# SAMPLE QUESTION PAPER 1
## Class – XII
## Mathematics

**Time allowed: 3hrs** **Maximum Marks: 100**

***General Instructions:***

*(i) All questions are compulsory.*
*(ii) The question paper consists of **26** questions divided into three Sections **A, B** and **C**.*
*(iii) Question No. **1** to **6** in Section **A**are Very Short Answer Type Questions carrying **one mark** each.*
*(iv) Question No. **7** to **19** in Section **B**are Long Answer I Type Questions carrying **four marks** each.*
*(v) Question No. **20** to **26** in Section **C**are Long Answer II Type Questions carrying **six marks** each.*
*(vii) Use of calculator is **not** permitted. You may ask for logarithmic tables, if required.*

### SECTION - A

1. Find the angle between the lines $2x = 3y = -z \text{ and } 6x = -y = -4z$.
2. Find $\lambda$ if the vectors $\hat{i} + 3\hat{j} + \hat{k},\ 2\hat{i} - \hat{j} - \hat{k},\ \lambda\hat{j} + 3\hat{k}$ are coplanar.
3. If m and n are the order and degree respectively of the differential equation $y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$, then write the value of m + n.
4. Find the differential equation representing the family of curves $xy = Ae^x + Be^{-x} + x^2$ where A and B are arbitrary constants.
5. Find unit vectors perpendicular to the vectors $\hat{i} + \hat{j} - \hat{k}$ and $-\hat{i} + 2\hat{j} + \hat{k}$.
6. On expanding by first row, the value of a third order determinant is $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$. Write the expression for its value on expanding by 2nd column, where $A_{ij}$ is the co-factor of element $a_{ij}$.

### SECTION - B

7. To promote the making of toilets for women an organisation tried to generate awareness through (i) House calls @ Rs. 50, (ii) Letters @ Rs 20 (iii) Announcements @ Rs. 40. The number of attempts made in three villages X, Y, Z are given below.

| | House Calls | Letters | Announcements |
|---|---|---|---|
| X | 400 | 300 | 100 |
| Y | 300 | 250 | 75 |
| Z | 500 | 400 | 150 |

Find the total cost incurred by the organisation for the three villages separately using matrices. Write one value generated by the organisation in the society.

8. Using properties of determinants, prove that: $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$.
9. Evaluate: $\int_0^1 \cot^{-1}(1 - x + x^2)\,dx$.
10. Let P (3, 2, 6) be a point in space and Q be a point on the line, $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 2\tilde{k})$, then find the value of $\lambda$ for which vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$.

**OR**

Find the distance of the point (-2, 3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

11. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-4}{5} = \frac{y+1}{5} = z$ intersect. Find the point of intersection also.
12. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = Sin^2\alpha$.

**OR**

Prove: $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$.

13. Evaluate:- $\int \frac{1}{x^4+1}dx$

**OR**

Evaluate: $\int (3-2x)\sqrt{2+x-x^2}\,dx$

14. Evaluate: $\int_{-1}^{3/2} |x\,Sin\,\bar{n}x|\,dx$

15. Find the inverse of matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, using elementary row transformations.

**OR**

If A = $\begin{bmatrix} -1 & 4 \\ 2 & -4 \end{bmatrix}$, Find $f(A)\,if\;f(x)=x^2-2x+3$.

16. If $y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{c}{x-c}+1$, prove that: $\frac{dy}{dx}=\frac{y}{x}\left[\frac{a}{a-x}+\frac{b}{b-x}+\frac{c}{c-x}\right]$

17. Show that $xy=ae^x+be^{-x}+x^2$ is a solution of differential equation $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy+x^2-2=0$.

18. Two dice are thrown simultaneously. Let X denote the number of Sixes, find the probability distribution of X. Also find mean and variance of X using probability distribution table.

**OR**

A man takes a step forward with probability 0.4 & backward with probability 0.6. Find the probability that at the end of 5 steps, he is one step away from the starting point.

19. Show that function $f(x)=2x-|x|$ is continuous but not differentiable at x = 0.

**SECTION - C**

20. Let X be a non-empty set. P(X) be its power set. Let * be an operation defined on elements of P(X) by

A * B = A $\cap$ B $\forall$ A, B $\in$ P (X).

Then,

(i) Prove that * is a binary operation in P (X).
(ii) Is * commutative?
(iii) Is * associative?
(iv) Find the identity element in P(X) w.r.t *
(v) Find all the invertible elements of P(X).
(vi) If o is another binary operation defined on P(X) as A o B = A $\cup$ B, then verify that o distributes itself over *.

OR

Let A = If $\{1,2,3,........9\}$ and R be the relation in A x A defined by (a, b) R(c, d) if a + d = b + c for a, b, c, d.

If $\in A$ . Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)].

21. Find the absolute maximum and absolute minimum values of the function $f$ given by $f(x)=Cos^2x+Sinx, x\in[0,\pi]$.

22. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P and solve it graphically.

23. A given quantity of metal is to be cast into a half circular cylinder (i.e. with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : (\pi + 2)$.

24. A letter is known as have come from either TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter has come from
(i) TATA NAGAR (ii) CALCUTTA

25. Sketch the graph of:

$$f(x) = \begin{cases} |x-2| + 2, & x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$$

Evaluate: $\int_0^4 f(x)\,dx$. What does the value of this integral represent on the graph?

26. Solve the following differential equation $(1 - x^2)\dfrac{dy}{dx} - xy = x^2$, given y = 2 when x = 0.

OR

Find the particular solution of the differential equation $xe^{y/x} - y\sin\left(\dfrac{y}{x}\right) + x\dfrac{dy}{dx}\sin\left(\dfrac{y}{x}\right) = 0$,

given that y = 0, when x = 1.

**SAMPLE PAPER 1**
**Solutions**

1. Equation of lines

$$\frac{x}{y_2}=\frac{y}{1/3}=\frac{z}{-1} \ and \ \frac{x}{1/6}=\frac{y}{-1}=\frac{z}{\frac{-1}{4}}$$

Angle between the lines

$$Cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\times\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$$Cos\theta=\left(\frac{\frac{1}{2}\times\frac{1}{6}+\frac{1}{3}(-1)+(-1)\left(\frac{-1}{4}\right)}{\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2+(-1)^2}\times\sqrt{\left(\frac{1}{6}\right)^2+(-1)^2+\left(\frac{1}{4}\right)^2}}\right)$$

$$Cos\theta=\left(\frac{\frac{1}{12}-\frac{1}{3}+\frac{1}{4}}{\sqrt{\frac{1}{4}+\frac{1}{9}+1}\times\sqrt{\frac{1}{36}+1+\frac{1}{16}}}\right)$$

$$Cos\theta=\left(\frac{\frac{1-4+3}{12}}{\sqrt{\frac{9+4+36}{36}}\times\sqrt{\frac{4+144+9}{144}}}\right)$$

$$Cos\theta=\left(\frac{\frac{0}{12}}{\sqrt{\frac{49}{36}}\times\sqrt{\frac{157}{14}}}\right)=0\,;$$

So, $\theta=90^0$

2. Let $\vec{a}=\hat{i}+3\hat{j}+\hat{k},\ \vec{b}=2\hat{i}-\hat{j}-\hat{k}\ and\ \vec{c}=\lambda\hat{j}+3\hat{k}$ for coplanar.

$$(\vec{a}\times\vec{b}).\vec{c}=0$$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & 1\\ 2 & -1 & -1\end{vmatrix}$$

$$\vec{a}\times\vec{b}=\hat{i}(-3+1)-\hat{j}(-1-2)+\hat{k}(-1-6)$$

$$\vec{a}\times\vec{b}=-2\hat{i}+3\hat{j}-7\hat{k}$$

And

$$(\vec{a}\times\vec{b}).\vec{c}=0$$

$$\left(-2\hat{i}+3\hat{j}-7\hat{k}\right).\left(\lambda\hat{j}+3\hat{k}\right)=0$$

$$0+3\lambda-21=0\,; \qquad 3\lambda=21; \qquad \lambda=7$$

3. $y-x\frac{dy}{dx}=\sqrt{a^2\left(\frac{dy}{dx}\right)^2+b^2}$

Squaring both sides

$$\left(y - x\frac{dy}{dx}\right)^2 = a^2\left(\frac{dy}{dx}\right)^2 + b^2$$

$$y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} = a^2\left(\frac{dy}{dx}\right)^2 + b^2$$

Order, m = 1, degree n = 2
So, m + n = 1+ 2= 3
m + n = 3

4. Given $xy = Ae^x + Be^{-x} + x^2$ ......(1)

Differentiating (1) twice w.r.t. x, we get

$x\frac{dy}{dx} + y.1 = Ae^x - Be^{-x}(-1) + 2x$ and

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx}.1 + \frac{dy}{dx} = Ae^x - Be^{-x}(-1) + 2x$$

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = Ae^x + Be^{-x} + 2$$

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = (xy - x^2) + 2$$ (using (1))

i.e. $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = xy - x^2 + 2$, which is the required differential equation.

5. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

Vector perpendicular to both $\vec{a} \,\&\, \vec{b} = \vec{a} \times \vec{b}$

Let $\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$

$= \hat{i}(1+2) - \hat{j}(1-1) + \hat{k}(2+1)$

$= 3\hat{i} + 3\hat{k}$

Unit vector $= \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{i} + 3\hat{k}}{\sqrt{3^2 + 3^2}} = \frac{3(\hat{i} + \hat{k})}{3\sqrt{2}} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

6. On expanding by 2nd Column

Let = $a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32}$

7. $A = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$, $B = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$

Total cost by the organisation for each village is represented by column matrix c.

$C = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$, Here C = AB

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 400\times50 + 300\times20 + 100\times40 \\ 300\times50 + 25\times20 + 75\times40 \\ 500\times50 + 400\times20 + 150\times40 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

There organisation is helping women in the society.

8. We have $\Delta = \begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$

Multiplying $R_1$, $R_2$, $R_3$ by a, b, c respectively and taking out a, b, c common from $C_1$, $C_2$ and $C_3$, we have

$$\Delta = \frac{abc}{abc}\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

Now applying $C_1 \to C_2 - C_1 \, and \, C_3 \to C_3 - C_1, we\, have$

$$\Delta = \begin{vmatrix} -bc & ab+bc+ac & ab+bc+ac \\ ab+bc & -(ab+bc+ac) & 0 \\ ac+bc & 0 & -(ac+bc+ac) \end{vmatrix}$$

By taking out (ab+bc+ac) common both from $C_2$ and $C_3$ we have

$$\Delta = (ab+bc+ac)^2\begin{vmatrix} -bc & 1 & 1 \\ ab+bc & -1 & 0 \\ ac+bc & 0 & -1 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$

$$\Delta = (ab+bc+ac)^2\begin{vmatrix} -bc & 1 & 1 \\ ab+bc & -1 & 0 \\ ac+bc & 0 & -1 \end{vmatrix}$$

Now, expanding along $C_3$, we have

$$\Delta = (ab+bc+ac)^2\left[-1.(-ac-ab-bc)\right]$$

$$= (ab+bc+ac)^3 = R.H.S$$

9. Let $I = \int_0^1 \cot^{-1}(1-x+x^2)\,dx$

$= \int_0^1 \tan^{-1}\frac{1}{1-x+x^2}\,dx$ $\quad \left[\because \cot^{-1}x = \tan^{-1}\frac{1}{x}\right]$

$= \int_0^1 \tan^{-1}\frac{x+(1-x)}{1-x(1-x)}\,dx$ $\quad [\because 1\, can\, be\, written\, as\, x+1-x]$

$= \int_0^1 \left[\tan^{-1}x + \tan^{-1}(1-x)\right]dx$ $\quad \left[\because \tan^{-1}\left\{\frac{a+b}{1-ab}\right\} = \tan^{-1}a + \tan^{-1}b\right]$

$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$

$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}[1-(1-x)]\,dx$ $\quad \left[\because \int_0^a f(x) = \int_0^a f(a-x)dx\right]$

$= 2\int_0^1 \tan^{-1}x\,dx = 2\int_0^1 \tan^{-1}x.1dx$

$= 2\left[\tan^{-1}1 - 0\right] - \int_0^1 \frac{2x}{1+x^2}\,dx = 2.\frac{\pi}{4} - [\log(1+x^2)]_0^1$

$= \frac{\pi}{2} - (\log 2 - \log 1) = \frac{\pi}{2} - \log 2$ $[\because \log 1 = 0]$

10. Line = $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 2\hat{k})$

Coordinates of point 'Q' on the line = $(1-3\lambda, -1+\lambda, 2+2\lambda)$

Given point P(3, 2, 6)

DR's of PQ = $1-3\lambda-3, -1+\lambda-2, 2+2\lambda-6$

= $-3\lambda-2, \lambda-3, 2\lambda-4$

Since PQ is ||to the plane x – 4y + 3z = 1.

Normal of given plane = $(\hat{i} - 4\hat{j} + 3\hat{k})$ as PQ $\perp n^{-1}$.

$1(-3\lambda-2) - 4(\lambda-3) + 3(2\lambda-4) = 0$

$-3\lambda - 2 - 4\lambda + 12 + 6\lambda - 12 = 0$

$-\lambda - 2 = 0; \ = \lambda = -2$

Co-ordinates of Q (7, -3, -2)

OR

Equation of plane:

$4x + 12y - 3z + 1 = 0$

So normal to the plane will

Be, $\vec{x} = 4\hat{i} + 12\hat{j} - 3\hat{k}$

And

Equation of line = $\frac{x+2}{3} = \frac{2(y+3/2)}{4} = \frac{(z+4/3)}{5/3} = \lambda$

Variable point 'Q' on the line = $\left(3\lambda - 2, 2\lambda - \frac{3}{2}, \frac{5}{3}\lambda - \frac{4/}{3}\right)$

Vector $\overrightarrow{PQ} = (3\lambda - 2 + 2)\hat{i} + (2\lambda - \frac{3}{2} - 3)\hat{j} + \left(\frac{5}{3}\lambda - \frac{4}{3} + 4\right)\hat{k}$

= $3\lambda\hat{i} + \left(2\chi\lambda - \frac{9}{2}\right)\hat{j} + \left(\frac{5}{3}\lambda + \frac{8}{3}\right)\hat{k}$

Since $\overrightarrow{PQ}$ ||Plane

$\overrightarrow{PQ}.\vec{x} = 0$

$\left(3\lambda\hat{i} + \left(2\lambda - \frac{9}{2}\right)\hat{j} + \left(\frac{5}{3}\lambda + \frac{8}{3}\right)\hat{k}\right).\left[4\hat{i} + 12\hat{j} - 3\hat{k}\right] = 0$

$12\lambda + 24\lambda - 54 - 5\lambda - 8 = 0$

$31\lambda - 62 = 0$; $\lambda = 2$

So Co-ordinates of Q = $\left[3(2) - 2, 2 \times 3 - \frac{3}{2}, \frac{5}{3} \times 2 - \frac{4}{3}\right]$

Q = (4, 5/2, 2)

Distance PQ = $\sqrt{(4+2)^2 + \left(\frac{5}{2} - 3\right)^2 + (2+4)^2}$

$\sqrt{36 + \left(\frac{-1}{2}\right)^2 + 36}$ = $\sqrt{72 + \frac{1}{4}} = \sqrt{\frac{289}{4}} = \frac{17}{2}$

Distance PQ= 8.5 units

11. For the lines to intersect shortest distance between them must be zero.

$l_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4};$ $l_2 = \frac{x-4}{5} = \frac{y-1}{+2} = \frac{z-0}{1}$

$a_1 = 1\hat{i} + 2\hat{j} + 3\hat{k}, \quad \mathbf{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$a_2 = 4\hat{i} + 1\hat{j} + 0\hat{k}, \quad b_2 = 5\hat{i} + 2\hat{j} + 1\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(-5) - \hat{j}(-18) + \hat{k}(-11); \qquad = -5\hat{i} + 18\hat{j} - 11\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} - 1\hat{j} - 3\hat{k}$

Distance d = $\left[ \dfrac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 - \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right] = \dfrac{|-15 - 18 + 33|}{\sqrt{(15)^2 + (18)^2 + (-11)^2}} = 0$

Since the shortest distance is zero for point of intersection

Let $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4} = \lambda \quad and \quad \dfrac{x-4}{5} = \dfrac{y-1}{2} = \dfrac{z}{1} = \mu$

$x = 2\lambda + 1 \qquad\qquad x = 5\mu + 4$

$y = 3\lambda + 2 \qquad\qquad y = 2\mu + 1$

$z = 4\lambda + 3 \qquad\qquad z = \mu$

On comparing

$2\lambda + 1 = 5\mu + 4 \qquad 3\lambda + 2 = 2\mu + 1 \qquad and \ 4\lambda + 3 = \mu$

$2\lambda - 5\mu = 3 \ \rightarrow (1) \qquad 3\lambda - 2\mu = -1 \rightarrow (2) \qquad 4\lambda - \mu = -3 \ \rightarrow (3)$

On solving $(1) \ \& (2)$

$\mu = -1, \quad \lambda = -1$

So point of intersection = (-1, -1, -1)

12. Let $Cos^{-1}\dfrac{x}{a} = A \ and \ Cos^{-1}\dfrac{y}{b} = B, \qquad it \ gives \ \dfrac{x}{a} = CosA, \ \dfrac{y}{b} = CosB$

So $A + B = \alpha$

Cos (A+B)= $Cos\alpha$

Cos A Cos B – Sin A Sin B = Cos$\alpha$

$$\frac{x}{a} \times \frac{x}{b} - \sqrt{1 - \frac{x^2}{a^2}} \times \sqrt{1 - \frac{y^2}{b^2}} = Cos\alpha$$

$$\frac{xy}{ab} - \sqrt{\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)} = Cos\alpha$$

$$\frac{xy}{ab} - Cos\alpha = \sqrt{1 - \frac{y^2}{b^2} - \frac{x^2}{a^2} + \frac{x^2y^2}{a^2b^2}}$$

Squaring both sides

$$\frac{x^2y^2}{a^2b^2} + Cos^2\alpha - \frac{2xy}{ab}Cos\alpha = 1 - \frac{y^2}{b^2} - \frac{x^2}{a^2} + \frac{x^2y^2}{a^2b^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}Cos\alpha = 1 - Cos^2\alpha$$

$$\frac{x^2}{a^2} - \frac{2xy}{ab}Cos\alpha + \frac{y^2}{b^2} = Sin^2\alpha$$

OR

Let $\dfrac{1}{2}Cos^{-1}\dfrac{a}{b} = A$

So, $\frac{a}{b} = Cos2A$

L.H.S $\tan\left(\frac{\pi}{4}+A\right)+\tan\left(\frac{\pi}{4}-A\right)$

$$= \frac{\tan\frac{\pi}{4}+\tan A}{1-\tan\frac{\pi}{4}\,.\,\tan A} + \frac{\tan\frac{\pi}{4}-\tan A}{1+\tan\frac{\pi}{4}\,.\,\tan A}$$

$$= \frac{\frac{1+\tan A}{1-\tan A}+\frac{1-\tan A}{1+\tan B}}{\frac{(1+\tan A)^2+(1-\tan A)^2}{1-\tan^2 A}}$$

$$= \frac{1+\tan^2 A+2\tan A+1+\tan^2 A-2\tan A}{1-\tan^2 A}$$

$$= \frac{2\left(1+\tan^2 A\right)}{1-\tan^2 A}; \quad = 2\times\frac{1}{Cos2A}; \quad = 2\times\frac{1}{a/b}; \quad = 2b/a$$

13. $\int \frac{1}{x^4+1}dx$

Dividing Numerator and Denominator by $x^2$

$$= \int \frac{\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\int \frac{\frac{2}{x}}{x^2+\frac{1}{x^2}}dx; \quad = \frac{1}{2}\int \frac{\frac{1}{x^2}+1+\frac{1}{x^2}-1}{x^2+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\left[\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2}dx - \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+2-2}dx\right]$$

$$= \frac{1}{2}\left[\int \frac{du}{(4)^2+(\sqrt{2})^2} - \int \frac{dv}{v^2-(\sqrt{2})^2}\right] \qquad \left\{\because \text{where } x-\frac{1}{x}=u \;\&\; x+\frac{1}{x}=v\right.$$

$$= \frac{1}{2}\left[\frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{4}{\sqrt{2}}\right)-\frac{1}{2\sqrt{2}}\log\left(\frac{v-\sqrt{2}}{v+\sqrt{2}}\right)\right]$$

$$= \frac{1}{2\sqrt{2}}\left[\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)-\frac{1}{2}\log\left(\frac{x^2+1-\sqrt{2x}}{x^2+1+\sqrt{2x}}\right)\right]+C$$

OR

$$\int (3-2x)\sqrt{2+x-x^2}\,dx$$

Let $3-2x = A\frac{d}{dx}\left(2+x-x^2\right)+B$

$3-2x = A(1-2x)+B$

$3-2x = -2Ax+A+B$

On comparing

=-2A = -2  A+B = 3

A = 1  1+B=3; = b = 2

$$= \int [1(1-2x)+2]\sqrt{2+x-x^2}\,dx$$

$$= \int (1-2x)\sqrt{2+x-x^2}\,dx + \int 2\sqrt{2+x-x^2}\,dx$$

$$= \int \sqrt{t}\ dt\ + 2\int \sqrt{2-\left(x^2-x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2\right)}\ dx$$

$$= \frac{(t)^{3/2}}{3/2} + 2\int \sqrt{\left(\frac{3}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}\,dx$$

$$= \frac{2}{3}\sqrt{2+x-x^2} + 2\times\frac{(x-1/2)}{2}\sqrt{\left(\frac{3}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} + \frac{(3/2)^2}{2} Sin^{-1}\left(\frac{x-\frac{1}{2}}{3/2}\right) + C$$

$$= \frac{2}{3}\sqrt{2+x-x^2} + \frac{2x-1}{2}\sqrt{2+x-x^2} + \frac{9}{8} Sin^{-1}\left(\frac{2x-1}{3}\right) + C$$

14. Here $f(x) = |x\sin\pi x| = \begin{cases} x\sin\pi x \ for\ -1\le x\le 1 \\ -x\sin\pi x\ for 1\le x\le \frac{3}{2} \end{cases}$

Therefore $\int_{-1}^{3/2} |x\sin\pi x|dx = \int_{-1}^{1} x\sin\pi x\,dx + \int_{1}^{\frac{3}{2}} -x\sin\pi x\,dx$

$$= \int_{-1}^{1} x\sin\pi x dx - \int_{1}^{\frac{3}{2}} -x\sin\pi x\,dx$$

Integrating both integrals on right hand side, we get

$$\int_{-1}^{3/2} |x\sin\pi x|\,dx = \left[\frac{-x\cos\pi x}{\pi}+\frac{\sin\pi x}{\pi^2}\right]^{1} - \left[\frac{-x\cos\pi x}{\pi}+\frac{\sin\pi x}{\pi^2}\right]_{1}^{\frac{3}{2}}$$

$$= \frac{2}{\pi} - \left[-\frac{1}{\pi^2}-\frac{1}{\pi}\right] = \frac{3}{\pi}+\frac{1}{\pi^2}$$

15. Let A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, $then\ A = I_3 A$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad Operate\ R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad Operate\ R_3 \to R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \qquad Operate\ R_3 \to R_3 + 5R_2$$

$$\begin{bmatrix}1&2&3\\0&1&2\\0&0&2\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\5&-3&1\end{bmatrix}A \qquad Operate\; R_1\to R_1-2R_2$$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&0&2\end{bmatrix}=\begin{bmatrix}-2&1&0\\1&0&0\\5&-3&1\end{bmatrix}A \qquad Operate\; R_2\to R_2-R_3$$

$$\begin{bmatrix}1&0&-1\\0&1&0\\0&0&2\end{bmatrix}=\begin{bmatrix}-2&1&0\\-4&3&-1\\5&-3&1\end{bmatrix}A \qquad Operate\; R_3\to \frac{1}{2}R_3$$

$$\begin{bmatrix}1&0&-1\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}-2&1&0\\-4&3&-1\\\frac{5}{2}&-\frac{3}{2}&\frac{1}{2}\end{bmatrix}A \qquad Operate\; R_1\to R_1+R_3$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-4&3&-1\\\frac{5}{2}&-\frac{3}{2}&\frac{1}{2}\end{bmatrix}A$$

$I_3$ = BA, where B = $\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-4&3&-1\\\frac{5}{2}&-\frac{3}{2}&\frac{1}{2}\end{bmatrix}$

Hence, $A^{-1}$ = B = $\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-4&3&-1\\\frac{5}{2}&-\frac{3}{2}&\frac{1}{2}\end{bmatrix}$

16. $y=\dfrac{ax^2}{(x-a)(x-b)(x-c)}+\dfrac{bx}{(x-b)(x-c)}+\left[\dfrac{c+x-c}{x-c}\right]$

$$y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{x}{x-c}\left[\frac{b}{x-b}+1\right]$$

$$y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\left[\frac{b+x-b}{x-b}\right]$$

$$y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{x^2}{(x-b)(x-c)}$$

$$y=\frac{x^2}{(x-b)(x-c)}+\left[\frac{a}{x-a}+1\right]=\frac{x^2}{(x-b)(x-c)}+\left[\frac{a+x-a}{x-a}\right]$$

$$y=\frac{x^3}{(x-a)(x-b)(x-c)}$$

Taking log on both sides

Log y = $\log x^3-\log(x-a)-\log(x-b)-\log(x-c)$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x^3}\times 3x^2-\frac{1}{x-a}-\frac{1}{x-b}-\frac{1}{x-c}$$

$$\frac{1}{y}\frac{dy}{dx}=\left[\frac{1}{x}-\frac{1}{x-a}+\frac{1}{x}-\frac{1}{x-b}+\frac{1}{x}-\frac{1}{x-c}\right]$$

$$\frac{dy}{dx}=y\left[\frac{x-a-x}{x(x-a)}+\frac{x-b-x}{x(x-b)}+\frac{x-c-x}{x(x-c)}\right]$$

$$\frac{dy}{dx}=\frac{y}{x}\left[\frac{a}{a-x}+\frac{b}{b-x}+\frac{c}{c-x}\right].$$

17. $xy=ae^x+be^{-x}+x^2$

$$x\frac{dy}{dx}+y\times 1=ae^x-be^{-x}+2x$$

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}\times 1+\frac{dy}{dx}=ae^x+be^{-x}+2$$

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=xy-x^2+2 \qquad [\{\because ae^x+be^{-x}=xy-x^2$$

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy+x^2-2=0$$

18. Clearly, X can take values 0, 1 and 2.
We have,

$P(X=0)=$ Probability of not getting six on any dice = $\frac{25}{36}$

$P(X=1)=$ Probability of getting one six = $\frac{10}{36}$

$P(X=2)=$ Probability of getting two sixes = $\frac{1}{36}$

Thus, the probability distribution of X is given by

| X: | 0 | 1 | 2 |
|---|---|---|---|
| P(X): | $\frac{25}{36}$ | $\frac{10}{36}$ | $\frac{1}{36}$ |

| $X_i$ | $P_i = P(X=x_i)$ | $P_i x_i$ | $P_i x_i^2$ |
|---|---|---|---|
| 0 | $\frac{25}{36}$ | 0 | 0 |
| 1 | $\frac{10}{36}$ | $\frac{10}{36}$ | $\frac{10}{36}$ |
| 2 | $\frac{2}{36}$ | $\frac{2}{36}$ | $\frac{4}{36}$ |
| | | $\sum p_i x_i=\frac{12}{36}$ | $\sum p_i x_i^2=\frac{14}{36}$ |

We have,

$$\sum p_i x_i=\frac{12}{36}=\frac{1}{3} \text{ and } \sum p_i x_i^2=\frac{7}{18}$$

E (X) = $\sum p_i x_i=\frac{1}{3}$

And, Var (X)= $\sum p_i x_i^2-\left(\sum p_i x_i\right)^2=\frac{7}{18}-\frac{1}{9}=\frac{5}{18}$

Hence, E(X) = $\frac{1}{3}$ $and\ Var(X) = \frac{5}{18}$

OR

The man is one step away from the starting point after 5 steps. This can happen in two ways

(i) He takes 5 steps forward and 2 steps backward

(ii) He takes 2 steps backward and 3 steps forward.

Probability of first case = ${}^5C_3(0.4)^3(0.6)^2$

Probability of second case = ${}^5C_2(0.4)^2(0.6)^3$

Hence total required probability

= ${}^5C_3(0.4)^3(0.6)^2 + {}^5C_2(0.4)^2(0.6)^3$

= $10\times(0.4)^3 x(0.6)^2 + 10\times(0.4)^2\times(0.6)^3$

= $10\times.16\times.36\times1.0 = 0.576$

19. $f(x) = 2x - |x| = \begin{cases} 2x - x & if\ x \geq 0 \\ 2x-(-x) & ifx < 0 \end{cases}$

= $\begin{cases} x & if\ x \geq 0 \\ 3x & if\ \ x < 0 \end{cases}$

Here $f(0) = 0$

$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} 3x = 3\times0$ and

$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} x = 0$

Since,

$\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = f(0)$

The function if continuous at x = 0 for differentiability test

L.H.D

$f^1(0) = \lim_{h\to0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to0} \frac{(-3h)-0}{-h} = \lim_{h\to0} \frac{-3h}{-k} = 3$

R.H.D

$f^1(0) = \lim_{h\to0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to0} \frac{h-0}{h} = 1$

Since, L.H.D + R.H.D f(x) is not differentiable at x = 0

20. Given

$A*B = A\cap B \qquad \forall A, B \in P(X)$

(i) Since, A, B $\in P(X)$

$\because A\cap B \in P(X) \quad \forall A, B \in P(X)$

Hence, A*B is a binary operation.

(ii) $A*B = A\cap B = B\cap A = B*A \qquad \forall A,\ B \in P(X)$

So, * is commutative.

(iii) $(A*B)*C = (A\cap B)*C = A\cap B\cap C = A\cap(B\cap C)$

= $A\cap(B*C) = A*(B*C) \forall A, B, C \in P(X)$

So, * is associative.

(iv) Let E be the identity element in P(X) with respect to *. Then

A * E = E * A = A For all A $\in$P(X)

$A\cap E = E\cap A = A$ For all $A\subset X,\ A\cap X = A$

= E = X

Thus, X is the identity element with respect to * on P(X).

(v) Let A be an invertible element of P(X) and let S be its inverse. Then,

A*S = X = S*A

$A\cap S = X = S\cap A$

= A = S = X [$\because A\subset X,\ S\subset X$ ]

Thus, X is the only invertible element of P(X) with respect to * and it is the inverse of itself.

(vi) We have, the relation

$A * B = A \cap B,\ A, B \in P(X)$

$A \text{ o } B = A \cup B,\ A, B \in P(X)$

Now, $A \text{ o } (B*C) = A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$= (A \text{ o } B) * (A \text{ o } C)$ (i)

Also, $(B*C) \text{ o } A = (B \cup C) \cup A = (B \cup A) \cap (C \cup A)$

$= (B \text{ o } A)*(C \text{ o } A)$ (ii)

Hence, from (i) and (ii), we conclude that 'o' distributes over'*'.

21. $f(x) = Cos^2 x + Sinx \qquad x \in [0, \pi]$

$f^1(x) = 2Cos \times (-Sinx) + Cosx$

$f^1(x) = Cos \times [-2Sinx + 1]$

For critical points, $f^1(x) = 0$

$Cosx[-2Sinx + 1] = 0$

$Cosx = 0,\ Sin\, x = \frac{1}{2}$

$x = \frac{\pi}{2},\ and\ x = \frac{\pi}{6}, \frac{5\pi}{6}$

All points are $\left[0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi\right]$

$f(0) = Cos^2 0 + Sin0 = 1 + 0 = 1$

$f\left(\frac{\pi}{6}\right) = Cos^2 \frac{\pi}{6} + Sin\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$

$f\left(\frac{\pi}{2}\right) = Cos^2 \frac{\pi}{2} + Sin\frac{\pi}{2} = 0 + 1 = 1$

$f\left(\frac{5\pi}{6}\right) = Cos^2 \frac{5\pi}{6} + Sin\left(\frac{5\pi}{6}\right) = \left(\frac{-\sqrt{3}}{2}\right)^2 + \frac{1}{2} = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$

$f(\pi) = Cos^2 \pi + Sin\pi = (-1)^2 + 0 = 1$

Hence, absolute maximum = $\left(\frac{5}{4}\right)$

Absolute minimum = 1

22. Let x chairs and y tables be made per week.

Now, according to question we have

| Machines | A | B | C |
|---|---|---|---|
| Chairs (x) | 2 | 1 | 1 |
| Tables (y) | 1 | 1 | 3 |

Here, total time available per week on machine A is 70 hours.

$2x + y \le 70$

That of machine B is 40 hours.

$x + y \le 40$

And machine C is available for 90 hours.

$x + 3y \le 90$

Its profit, Z 30 x+ 60 y

So, the LPP is $Z_{max} = 30x + 60y$

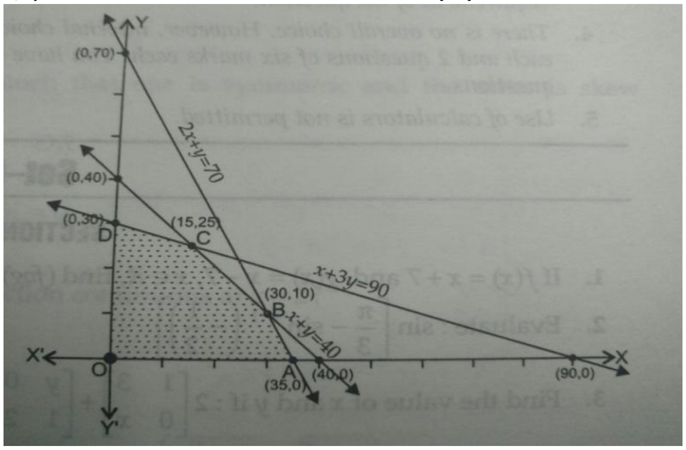
Subject to the constraints

$2x + y \le 70 \qquad (i)$

$x + y \le 40 \qquad$ (ii)

$x + 3y \le 90$ (iii)

x, y $\ge 0$ (iv)

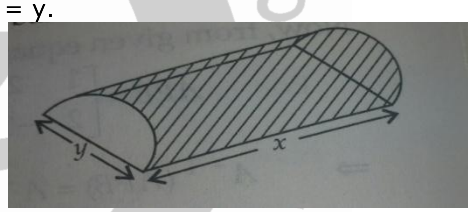



On plotting (i) to (iv), and shading according to constraints we have
Here, OABCD is the required feasible region (show in figure) which is bounded.
Now, we find the value of Z at each corner point.

| Corner point | Z |
|---|---|
| A (35,0) | 1050 |
| B (30,10) | 1500 |
| C (15, 25) | 1950 |
| D (0, 30) | 1800 |

So, the furniture firm has maximum profit of Rs. 1950, if the firm made 15 chairs and 25 tables per week.

23. A given quantity of metal is cast into a half cylinder with a rectangular base and semicircular ends.
Let the length of rectangular base = x and its breadth = y.




Diameter of the semicircle = y

Radius = $\frac{y}{2}$.

Let the volume of metal = V, which is known and is constant.

$$\frac{1}{2}\pi\left(\frac{y}{2}\right)^2 . x = V \qquad \Rightarrow \frac{\pi x y^2 8}{} = V \qquad x = \frac{8V}{\pi y^2} \qquad ........(i)$$

To minimise the total surface area, S, we have

$$S = \pi\left(\frac{y}{2}\right)x + xy + \pi\left(\frac{y}{2}\right)^2 = \left(\frac{\pi}{2}+1\right)xy + \frac{\pi}{4}y^2$$

$$= \left(\frac{\pi}{2}+1\right)y.\frac{8V}{\pi y^2} + \frac{\pi}{4}y^2 = \left(\frac{\pi}{2}+1\right).\frac{8V}{\pi y} + \frac{\pi}{4}y^2$$

$$\frac{dS}{dy} = \left(\frac{\pi}{2}+1\right)\frac{8V}{\pi}.\left(-\frac{1}{y^2}\right) + \frac{\pi}{4}2y = \frac{\pi}{2}y - \left(\frac{\pi}{2}+1\right)\frac{8V}{\pi y^2}$$

$$\frac{dS}{dy} = 0 \qquad \Rightarrow \frac{\pi}{2}y = \left(\frac{\pi}{2}+1\right)\frac{8V}{\pi y^2} \qquad \Rightarrow \pi^2 y^3 = 8V(\pi+3)$$

$$\Rightarrow y^3 = \frac{8V(\pi+2)}{\pi^2}$$

$$\frac{dS}{dy} = \frac{\pi}{2}y - \left(\frac{\pi}{2}+1\right)\frac{8V}{\pi y^2} \qquad \Rightarrow \frac{d^2S}{dy^2} = \frac{\pi}{2} - \left(\frac{\pi}{2}+1\right)\frac{8V}{\pi}\left(\frac{-2}{y^3}\right)$$

When $\pi^2 y^3 = \frac{8V(\pi+2)}{\pi^2}$

$$\frac{d^2S}{dy^2} = \frac{\pi}{2} - \left(\frac{\pi}{2}+1\right).\frac{8V}{\pi}\frac{-2.\pi^2}{8V.(\pi+2)}$$

$$= \frac{\pi}{2} + \frac{\pi+2}{2}.\frac{2\pi}{\pi+2} = \frac{\pi}{2} + \pi = \frac{3\pi}{2} > 0$$

Total surface area, S, is minimum

When 8v = $\frac{\pi^2 y^3}{\pi+2}$ *and*

$$x = \frac{8V}{\pi y^2} = \frac{1}{xy^2}.\left(\frac{\pi^2 y^3}{\pi+2}\right) = \frac{\pi}{\pi+2}y \qquad [from(i)]$$

$$\Rightarrow \frac{x}{y} = \frac{\pi}{\pi+2} \qquad \Rightarrow x : y = \pi : (\pi+2)$$

Ratio of length of the cylinder to its diameter = $\pi$ : ( $\pi$ +2).

24. Let $E_1$ be the event that the letter came from Calcutta and $E_2$ be the event that the letter came from Tatanagar. Let A denote the event that two consecutive letters visible on the envelope are TA.

Since the letters have come either from Calcutta or Tatanagar, therefore, $P(E_1) = \frac{1}{2} = P(E_2).$

If $E_1$ has occurred, then it means that the letter came from Calcutta. In the word CALCUTTA there are 7 consecutive alphabet i.e., {CA, AL, LC, CU, UT, TT and TA} and TA occurs only one time.

Therefore, $P(A/E_1) = \frac{1}{7}$

If $E_2$ has occurred, then the letter came from TATANAGAR. In the word TATANAGAR there are 8 consecutive letters i.e., {TA, AT, TA, AN, NA, AG, GA, AR} in which TA occurs twice.

Therefore, $(A/E_2) = \frac{2}{8} = \frac{1}{4}$

By Baye's theorem:

(i) $$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)} = \frac{\frac{1}{2}\times\frac{1}{7}}{\frac{1}{2}\times\frac{1}{7}+\frac{1}{2}\times\frac{1}{4}} = \frac{4}{11}$$

(ii) $$P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)} = \frac{\frac{1}{2}\times\frac{1}{4}}{\frac{1}{2}\times\frac{1}{7}+\frac{1}{2}\times\frac{1}{4}} = \frac{7}{11}$$

25. Given, $$f(x) = \begin{cases} |x+2|+2, & x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$$

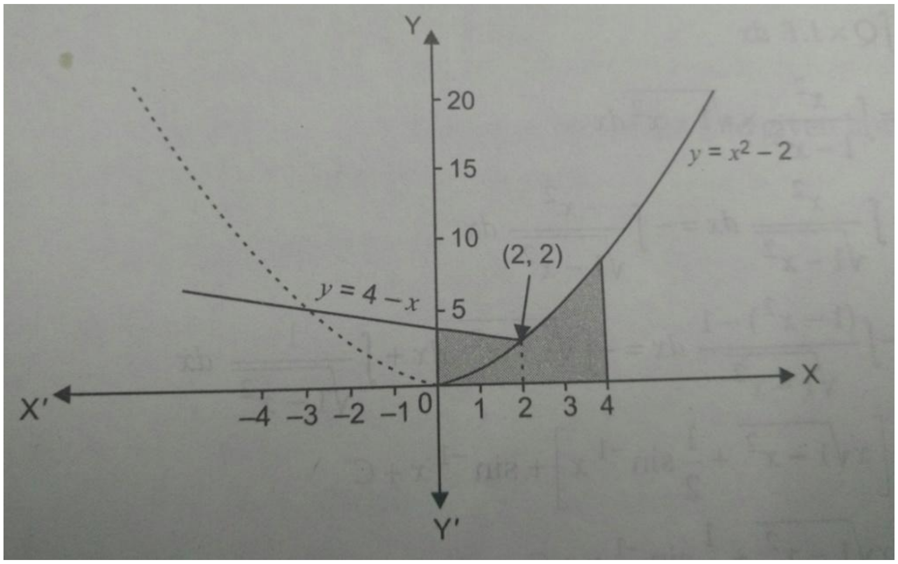
For $x \le 2,$ $f(x) = |x-2| + 2 = -(x-2) + 2 = 4 - x$

| | | |
|---|---|---|
| X | 2 | 1 |
| F(x) | 2 | 3 |

For $x > 2,$ $f(x) = x^2 - 2$

| | | |
|---|---|---|
| X | $\pm 3$ | $\pm 4$ |
| Y | 7 | 14 |

The graph of the function is shown in the figure.




Now, $\int_0^4 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx$

$$= \int_0^2 (|x-2|+2)dx + \int_2^4 (x^2-2)dx$$

$$= \int_0^2 -(x-2)+2)dx + \left(\frac{x^3}{3} - 2x\right)^4$$

$$= \left[\frac{-x^2}{2} + 4x\right]^2 + \left[\frac{64}{3} - 8 - \frac{8}{3} + 4\right]$$

$$= 2 + 8 + \frac{56-12}{3} = 6 + \frac{44}{3} = \frac{18+44}{3} = \frac{62}{3} \, sq.units$$

The above value of the integral represent the area of the shaded region on the graph

26. $(1-x^2)\frac{dy}{dx} - xy = x^2,$ given y = 2, x = 0

$$\Rightarrow \frac{dy}{dx} - \frac{x}{1-x^2}.y = \frac{x^2}{1-x^2}$$

It is linear equation of the form

$$\frac{dy}{dx} + Py = Q$$

Here, $P = \frac{-x}{1-x^2}$, $Q = \frac{x^2}{1-x^2}$

Now, I.F= $e^{\int Pdx} = e^{\int \frac{-x}{1-x^2}dx} = e^{\frac{1}{2}\int \frac{-2}{1-x^2}dx} = e^{\frac{1}{2}\log(1-x^2)} = e^{\log}\sqrt{1-x^2} = \sqrt{1-x^2}$

$\therefore$ *Its Solution is*

$$y \times I.F = \int Q \times I.F \; dx$$

$$\Rightarrow y\sqrt{1-x^2} = \int \frac{x^2}{1-x^2} \times \sqrt{1-x^2}\,dx$$

$$\Rightarrow y\sqrt{1-x^2} = \int \frac{x^2}{\sqrt{1-x^2}}dx = -\int \frac{-x^2}{\sqrt{1-x^2}}dx$$

$$= -\int \frac{(1-x^2)-1}{\sqrt{1-x^2}}dx = -\int \sqrt{1-x^2}\,dx + \int \frac{1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow y\sqrt{1-x^2} = -\left[x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right] + \sin^{-1}x + C$$

$$\Rightarrow y\sqrt{1-x^2} = -x\sqrt{1-x^2} + \frac{1}{2} + \sin^{-1}x + C$$

Now, when x = 0, y = 2 we have

$$\Rightarrow 2 = C$$

∴ Required solution is

$$y\sqrt{1-x^2} = -x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x + 2$$