

SAMPLE QUESTION PAPER 4

Class – XII MATHEMATICS

Time allowed: 3hrs

Maximum Marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) The question paper consists of **26 questions** divided into three Sections **A, B and C**.
- (iii) Question No. **1 to 6** in Section A are Very Short Answer Type Questions carrying **one mark** each.
- (iv) Question No. **7 to 19** in Section B are Long Answer I Type Questions carrying **four marks** each.
- (v) Question No. **20 to 26** in Section C are Long Answer II Type Questions carrying **six marks** each.
- (vi) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- (vii) Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION A

Question number 1 to 6 carry 1 mark each

1. The position vectors of points A and B are \vec{a} and \vec{b} respectively
P divides AB in the ratio 3 : 1 and Q is mid-point of AP. Find the position vector of Q.
2. Find the area of the parallelogram, whose diagonals are $\vec{d}_1 = 5\hat{i}$ and $\vec{d}_2 = 2\hat{j}$
3. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane.
4. If $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$ Write the cofactor of a_{32} (the element of third row and 2nd column).
5. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m+n.
6. Write the differential equation representing the curve $y^2 = 4ax$, where a is an arbitrary constant.

SECTION B

Question numbers 7 to 19 carry 4 marks each

7. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastelsheets made by them using recycled paper, at the rate of Rs. 20, Rs.15 and Rs. 5 per unit respectively. School A sold 25 paper-bags 12 scrap-books and 34 pastel sheets. School B sold 22 paper-bags, 15 scrapbooks and 28 pastel-sheets while school C sold 26 paper-bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school. By such exhibition, which values are inculcated in the students?
8. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$, then show that $A^2 - 4A + 7I = 0$. Using this result also calculate A^3

9. If x, y, z are in GP, then using properties of determinants, show that

$$\begin{vmatrix} px + y & x & y \\ px + z & y & z \\ 0 & px + y & px + z \end{vmatrix} = 0, \text{ where } x \neq y \neq z \text{ and } p \text{ is any real number.}$$

10. Evaluate $\int_{-1}^1 |x \cos \pi x| dx$

11. Evaluate $\int \frac{1 + \sin 2x}{1 + \cos 2x} \cdot e^{2x} dx$.

12. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

13. Find the position vector of the foot of perpendicular drawn from the point $P(1, 8, 4)$ to the line joining $A(0, -1, 3)$ and $B(5, 4, 4)$. Also find the length of this perpendicular.

14. Show that $(|\vec{b}| \vec{a} + |\vec{a}| \vec{b}) \cdot (|\vec{b}| \vec{a} - |\vec{a}| \vec{b}) = 0$

15. Prove that $2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$

16. If $x^y + y^x + x^x = a^b$. Find $\frac{dy}{dx}$

17. Find the distance between the lines l_1 and l_2 given by:

$$\vec{r} = (i + 3j - 2k) + \alpha(2i - 3j + k)$$

$$\vec{r} = (2i + 4j - k) + \mu(2i - 3j + k)$$

18. Verify Rolle's theorem for $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$

19. Integrate $\int \frac{dx}{x(x^4 - 1)}$

SECTION C

Question numbers 20 to 26 carry 6 marks each

20. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $a, b, c, d \in A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2, 5)]$.

21. 40% students of a college reside in hostel and the remaining resides outside. At the end of year, 50% of the hoteliers got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hotelier?

22. A man rides his motorcycle at the speed of 50 km/h. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 80 km/h, the petrol cost increases to Rs. 3 per km. He has at most Rs. 120 to spend on petrol and one hour's time. Using LPP find the maximum distance he can travel.

23. A jet of enemy is flying along the curve $y = x + 2$ and a soldier is placed at the point $(3, 2)$. Find the minimum distance between the soldier and the jet.

24. Obtain the differential equation of all circles of radius r .

25. Compute, using integration, the area bounded by the lines

$$x + 2y = 2, \quad y - x = 1 \text{ and } 2x + y = 7$$

26. Show that the lines $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \beta(-3\hat{i} + \hat{j} + 5\hat{k})$

$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$ are coplanar. Also, find the equation of the plane containing these lines.