

SAMPLE QUESTION PAPER 3

Class – XII MATHEMATICS

Time allowed: 3hrs

Maximum Marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) The question paper consists of **26 questions** divided into three Sections **A, B and C**.
- (iii) Question No. **1 to 6** in Section A are Very Short Answer Type Questions carrying **one mark** each.
- (iv) Question No. **7 to 19** in Section B are Long Answer I Type Questions carrying **four marks** each.
- (v) Question No. **20 to 26** in Section C are Long Answer II Type Questions carrying **six marks** each.
- (vi) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- (vii) Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION A

Question number 1 to 6 carry 1 mark each

1. Suppose X is a 2x3 matrix, Z is a 5x3 matrix. Find the order of Y such that both X Y and Y Z are well defined.
2. Find the area of the triangle with vertices at the points (1,0),(6,0),(4,3).
3. Find x such that $\begin{vmatrix} 3 & X \\ X & 1 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix}$
4. Give example of a function which is neither one-one nor onto.
5. Calculate the direction cosines of the vector $a = 3i - 5j + 5k$
6. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1 L_2) : L_1 \text{ is parallel to } L_2\}$. Is L reflexive?

SECTION B

Question numbers 7 to 19 carry 4 marks each

7. Using properties of determinants prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

8. A poisonous substance is dropped in a lake next to a village. The waves move in circles at a speed of 2cm per second. At the instant when radius of the circular wave is 14cm, evaluate how fast the enclosed area is increasing. Discuss two harmful consequences of polluting water bodies.
9. Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are onto, then $g \circ f: A \rightarrow C$ is onto
10. Verify Rolle's theorem for $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$
11. Evaluate $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$
12. Show that the points A, B and C with position vectors

$\vec{a} = 3i - 4j$, $\vec{b} = 2i - j + k$ and $\vec{c} = i - 3j - 5k$ form the vertices of a right angled triangle.

13. The probability of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability :

(a) problem is solved

(b) exactly one of them solves the problem.

14. Find all points of discontinuity of the function f where f is defined by:

$$f(x) = \begin{cases} x^3 - x + 1, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 3x + 2, & x \geq 3 \end{cases}$$

15. Solve the differential equation $x \frac{dx}{dy} - y + x \operatorname{cosec} \left(\frac{y}{x} \right) = 0$, $y(1) = 0$

16. Show that $(|\vec{b}|\vec{a} + |\vec{a}|\vec{b}) \cdot (|\vec{b}|\vec{a} - |\vec{a}|\vec{b}) = 0$

17. Integrate $\int \frac{dx}{x(x^4-1)}$

18. Find the distance between the lines l_1 and l_2 given by:

$$\vec{r} = (i + 3j - 2k) + \alpha(2i - 3j + k)$$

$$\vec{r} = (2i + 4j - k) + \mu(2i - 3j + k)$$

19. Find the vector equation of the plane passing through the intersection of the planes

$$\vec{r} \cdot (2i + 2j - 3k) = 7, \quad \vec{r} \cdot (2i + 4j + 3k) \text{ and the point } (2, 1, 3)$$

SECTION C

Question numbers 20 to 26 carry 6 marks each

20. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

21. Solve the following system of equations using matrix method

$$\frac{3}{x} - \frac{2}{y} + \frac{3}{z} = 8$$

$$\frac{2}{x} + \frac{1}{y} - \frac{1}{z} = 1$$

$$\frac{4}{x} - \frac{3}{y} + \frac{3}{z} = 4$$

22. A factory can hire two tailors A and B in order to stitch pants and shirts. Tailor A can stitch 6 shirts and 4 pants in a day. Tailor B can stitch 10 shirts and 4 pants in a day. Tailor A charges 15 per day and tailor B charges 20 per day. The factory has to produce minimum 60 shirts and 32 pants. State as a linear programming problem and minimize the labour cost.

23. Find the area of the region included between the two parabolas y

24. Bag X contains 2 white and 3 red balls. Bag Y contains 5 white and 4 red balls. Bag Z contains 2 white and 3 red balls. A ball is drawn at random from one of the bags and it is found to be red. What is the probability that it is drawn from bag Y?

25. If $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$, find $\frac{dx^2}{dy^2}$

26. Integrate $\int_0^x \frac{x dx}{4 \cos^2 x + 9 \sin^2 x}$