Mathematics

Class - XII

Chapter Assignments

Chapter 1

Relations and Functions

1 mark Questions

1. If R= {(a, a^3): a is a prime number less than 5} be a relation. Find the range of R.
2. If *f*:{1,3,4}→{1,2,5} and g: {1,2,5} →{1,3} given by f = {(1,2), (3,5), (4,1)} and *g*= {(1,3), (2,3), (5,1)}. Write down *gof*.
3. If f: R →R is defined by f(X) = 3x+2, then define f [f(x)].
4. What is the range of the function
   a. f (x)= $\frac{|x-1|}{x-1}$, x≠1?
5. If f is an invertible function, defined as
   a. f(x)= $\frac{3x-4}{5}$, then write $f^{-1}(x)$.
6. If f:R→R and g:R→R are given by
   a. F(X) + sin x and g(X)= $5x^2$, then find gof(x).
7. State whether the function f: N→N given by f(x)= 5x is injective, surjective or both.
8. If R= {(x, y): x+2y=8} is a relation on N, then write the range of R.
9. Let *: R×R→R given by (a,b)→ a+$4b^2$ be a binary operation. Compute (-5)*(2*0).
10. Let * is a binary operation on N given by a*b = LCM (a,b) for all a,b € N. Find 5*7.
11. Let * is a binary operation on set of integers I, defined by a*b = 2a+b-3. Find the value of 3*4.
12. If the binary operation * on set of integers Z is defined by a*b= a+ $3b^2$, then find the value of 2*4.
13. Let * is the binary operation *, defined on Q, is defined as a*b= 2a+b-ab, for all as b€Q. Find the value of 3*4.
14. If * is a binary operation on set Q of a rational numbers defined as a*b=$\frac{ab}{5}$. Write the identity for *, if any.

4 marks Questions

1. If f: W→W, is defined as f(x) = x-1, if x is odd and f(x) = x+1, if x is even. Show that f is invertible. Find the inverse of *f*, where W is the set of all whole numbers.
2. If R is a relation defined on the set of natural number N as follows:
   a. R= {(x,y), x€N, Y€N and 2x+y +24}, then find the domain and range of the relation R. Also, find if R is an equivalence relation or not.
3. If A= R-{3} and B=R-{1}. Consider the function f:A→B defined by f(x)= $\frac{x-2}{x-3}$ for all x€A. Then show that f is bijective. Find $f^{-1}(x)$.

4. Consider f: $R_+ \rightarrow [4, \infty]$ fiven by f(x)= x^2 +4. Show that f is invertible with inverse f^{-1} of *f* given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.
5. If A= {1,2,3….,9}and R be the relation in A×A defined by (a,b) R(c,d). If a+d=b+c for (a,b), (c,d) in A×A. Prove that R is an equivalence relation. Also, obtain the equivalence class [(2,5)]
6. Show that the relation S in the set R of real numbers defined as, S= {(a,b): a,b€R and a≤b^3} is neither reflexive nor symmetric nor transitive.
7. Show that the relation S in set
    a. A= {x€Z: 0≤x≤2} given by S= {(a,b): a,b €Z, |a-b| is divisible by 4} is an equivalence relation. Find the set of all elements related to A.
8. Show that f:N→N, given by
    a. f (x)= $\begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$
    b. is bijective(both one-one and onto)
9. Show that a function f:R→R given by
    a. f(x)= ax+b, a,b €R, a≠0 is a bijective.
10. Show that relation R in the set of real numbers, defined as R= {(a,b): a≤b^2} is neither reflexive, nor symmetric nor transitive.
11. If the function f:R→R is given by
    a. f(x)= $\frac{x+3}{3}$ and g:R→R is given by g(x) = 2x-3, then find
12. fog
13. gof. Id f^{-1} =g?
14. If S is the set of all rational numbers except 1 and * be defined on S by a*b= a+b-ab, fo all a,b€s.
    a. Prove that
15. * is a binary operation on S.
16. * is commutative as well as associative
17. Consider the binary operations * : R×R→R and o: R×R→R defined as a*b + |a-b| and a o b= a. For all a,b €R. Show that * is associative but not commutative.
18. A binary operation * on the set {0,1,2,3,4,5} is defined by a*b= $\begin{cases} a+b, if\ a+b<6 \\ a+b-6, if\ a+b \geq 6 \end{cases}$
19. If * is a binary operation on set Q of rational numbers such that a*b= $(2a-b)^2$,a,b€Q. Find 3*5, 5*3. Is 3*5=5*3.
20. If * is a binary operation on Q, defined by a*b= 3ab/5. Show that * is commutative as well as associative. Also, find its identity, if it exists.

Chapter 2

Inverse Trigonometric Functions

1 mark Questions

1. If sin (sin$^{-1}\frac{1}{5}$ + cos^{-1} x) =1, then find the value of x.
2. If tan^{-1}+ tan^{-1}y=$\frac{\pi}{4}$; xy<1, then write the value of x+y+xy.
3. If sin $(\sin^{-1} + \cos^{-1}x) = 1$, then find the value of x.
4. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}; xy < 1$, then write the value of x + y + xy.
5. Write the principal value of $\cos^{-1}[\cos(80)^0]$.
6. Write the principal value of $\left[\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(-\frac{1}{2}\right)\right]$.
7. Write the value of tan $\left(2\tan^{-1}\frac{1}{5}\right)$.
8. Write the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$.
9. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.
10. Write the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.
11. Write the value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$?
12. Write the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.
13. Using the principal values, find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.
14. If $\tan^{-1}(\sqrt{3}) + \cot^{-1}x = \frac{\pi}{2}$, then find the value of x.
15. Using the principal values, evaluate$\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right)$.

4 marks Questions

1. Solve following equation for x.
2. $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$
3. Solved for x, 2 $\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$.
4. Solved for x, $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$.
5. Prove that
6. $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}; x \in \left(0, \frac{\pi}{4}\right)$
7. Prove that
8. $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right] = \frac{\pi}{4} = \frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}} \leq x \leq 1$
9. Prove that
10. $\cos^{-1}(x) + \cot^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{4}$.

11. Prove the following:

12. $\cot^{-1}\left[\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{\pi}{4}; x\in\left(0,\frac{\pi}{4}\right)$.

13. Solve for x, $\tan^{-1}3x+\tan^{-1}2x=\frac{\pi}{4}$.

14. Prove that

15. $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}=\frac{\pi}{4}$.

16. Prove that

17. $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$.

18. Find the value of $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{31}{17}\right)$.

19. Prove that

20. $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$.

21. Solve for x, $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right), x\in(0,1)$.

22. Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$.

23. Solve for x, $\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{4}, \sqrt{6}>x>0$,

24. Prove that

25. $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.

26. Solve for x,

27. $\tan^{-1}\left(\frac{1+x}{1-x}\right)=\frac{\pi}{4}+\tan^{-1}x, 0<x<1$.

Chapter 3

Matrices

1 mark Questions

1. Solve the following matrix equation for x.
   a. $[x\text{-}1]\begin{bmatrix}1 & 0\\ -2 & 0\end{bmatrix}=0$
2. If A is a square matrix such that $A^2=A$, then write the value of $7A-(I+A)^3$, where I is an identity matrix.
3. If $\begin{bmatrix}a+4 & 3b\\ 8 & -6\end{bmatrix}\begin{bmatrix}2a+2 & b+2\\ 8 & a-8b\end{bmatrix}$, write the value of a-2b.
4. The elements a_{ij} of a 3×3 matrix are given by $a_{ij}=\frac{1}{2}|-3i+j|$. Write the value of element a_{32}.
5. If $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$, then find the value of (x+y)
6. If matrix A $\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$ and A^2 =pA, then write the value of p.
7. Simplify
   a. $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$
8. Find the value of y-x from following equation
   a. $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$
9. For a 2×2 matrix, A= $[a_{ij}]$ whose elements are given by a_{ij}= i/j, write the value of a_{12}.
10. From the following matrix equation, find the value of x
    a. $\begin{bmatrix}x+4 & 3y4\\ -5 & \end{bmatrix}=\begin{bmatrix}3 & 4\\ -5 & 6\end{bmatrix}$
11. If A = $\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then for what value of α, A is an identity matrix?
12. If A is a matrix of order 3×4 and B is a matrix of order 4×3, then find order of matrix (AB).
13. If $\begin{bmatrix}2x+y & 3y\\ 0 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\ 0 & 4\end{bmatrix}$, then find the value of x.
14. Find the value of x, if
    a. $\begin{bmatrix}3x+y & -y\\ 2y-x & 3\end{bmatrix}=\begin{bmatrix}1 & 2\\ -5 & 3\end{bmatrix}$
15. Find x and y, if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$
16. Write 2×2 matrix which is both symmetric and skew=symmetric matrices.
17. For what value of x, is the matrix
    a. A= $\begin{matrix}0 & 1 & -2\\ -1 & 0 & 3\\ x & -3 & 0\end{matrix}$ a skew-symmetric matrix?
18. If A^T= $\begin{matrix}3 & 4\\ -1 & 2\\ 0 & 1\end{matrix}$ and B= $\begin{matrix}-1 & 2 & 1\\ 1 & 2 & 3\end{matrix}$, then find A^T-B^T.
19. If matrix A= [1 2 3], then write AA'.

a. Ques 20, pg 50. Ques 11, pg 50

4 marks Questions

1. If A=$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the value of A^2-3a+ 2I.
2. For the following matrices A and B, verify that $[AB]' = B'A'$; A=$\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, B= [-1 2 1].
3. Express the following matrix as a sum of a symmetric and a skew symmetric matrices and verify your result.
4. $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$
5. Use elementary colum operations $C_2 \rightarrow C_2 - 2C_1$ in the matrix equation
6. $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$
7. Using elementary row transformations, find inverse of matrix A= $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$
8. 6 marks Questions
9. $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, Using elementary row transformations, find inverse of matrix.
10. $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$, Using elementary row transformations, find inverse of matrix.
11. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$, Using elementary row transformations, find inverse of matrix.

Chapter 4

Determinants

1 mark Questions

1. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix}=\begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then write the value of x.
2. If A is a 3×3 matrix, |A|≠0 and |3A|=k|A|, then write the value of k.
3. Find (adj A), if A= $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$.
4. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix}=\begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then find the value of x.
5. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32}.A_{32}$.
6. If $\Delta=\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, then write the minor of the element a_{23}.
7. If the determinant of matrix A of order 3×3 is value of 4, then write the value of |3A|.
8. For what value of x, the matrix $\begin{bmatrix} 6 & -x & 4 \\ 3 & -x & 1 \end{bmatrix}$ is a singular matrix.
9. What is the value of determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$?
10. Find the minor of the element of second row and third column in the determinant
    a. $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
11. If A=$\begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$, then find |adj A|.
12. If |A|=2, where A is a 2×2 matrix, then find |adj A|.
13. If A is a non-singular matrix of order 3 and |adj A|= $|A|^k$, then what is the value of k?
14. Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.
15. Evaluate$\begin{vmatrix} 2cos\theta & -2sin\theta \\ sin\theta & cos\theta \end{vmatrix}$.
16. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix}$=0, then find the value of x.
17. Prove the following, using properties of determinants
    a. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$
18. Write the value of determinant $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$
19. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

4 marks Questions

1. Using properties of determinants, prove that

   a. $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

2. Using properties of determinants, prove that

   a. $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

3. Using properties of determinants, prove that:

   a. $\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2$

4. Using properties of determinants, prove that

   a. $\begin{vmatrix} 1 & x & x2 \\ x2 & 1 & x \\ x & x2 & 1 \end{vmatrix} += (1-x^3)^2$

5. Using properties of determinants, prove that

   a. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$

6. Using properties of determinants, prove that

   a. $\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx)$

7. Using properties of determinants, prove that

   a. $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha 2 & \beta 2 & \gamma 2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

8. Using properties of determinants, prove that

   a. $\begin{vmatrix} -a2 & ab & ac \\ ba & -b2 & bc \\ ca & cb & -c2 \end{vmatrix} = 4a^2b^2c^2.$

9. Using properties of determinates, solve the following for x.

   a. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

10. Using properties of determinants solve the following for x.

    a. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

11. Prove, using properties of determinants

    a. $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

12. Prove that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1.$

13. Using properties of determinants, prove that

a. $\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$ = $(x+y+z)^3$

14. Prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c\_a \\ b+c & c+a & a+b \end{vmatrix}$ = $a^3 + b^3 + c^3 - 3abc$.

6 marks Questions

1. Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award Rs. X each, Rs y each and Rs Z each for the three respective values of 3,2, and 1 students respectively with a total award money of Rs. 1000. School Q wants to spend Rs. 1500 to award its 4,1 and 3 students on the respective values (by giving as before). If the total amount of award for one prize on each value is Rs. 600, using matrices, find the award money for each value.
2. Apart from the above three values, suggest one more value for awards.
3. A total amount of Rs. 7000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8%, and 81/2% respectively. The total annual interest from these three accounts is Rs. 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices.
4. Using matrices solve the following system of linear equations.
5. x+y-z=3
6. 3x+4y-5z=-5
7. And 2x-y=3z=12
8. If A= $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, then find A^{-1} and hence solve the following equations
9. X+2y+z=4
10. -x+y+z=0
11. X-3y+z=4
12. Determine the product of $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and then use to solve the system of equations
13. x-y+z=4
14. x-2y-2z=9
15. 2x+y+3z=1
16. Using matrix method, solve the following system of equations.
17. X+2y+z=7
18. X+3z=11
19. 2x-3y=1
20. Using matrices, solve the following system of equations

21. 2x+y+z=7
22. x-y-z=-4
23. 3x+2y+z=10
24. If A=$\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1}. Hence, solve the following system of equations
25. 3x+2y+z=6
26. 4x-y+2z=5
27. 7x+3y-3z=7
28. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.
29. Using matrices, solve the following system of equations
30. x-y+2z=7
31. 3x+4y-5z+-5
32. 2x-y+3z=12.

Chapter 5

Continuity and Differentiability

1 mark Questions

1. If y=sin^{-1}{z$\sqrt{1-x}-\sqrt{x}\sqrt{1-x2}$} and o<x<1, then find$\frac{dy}{dx}$.
2. If $e^X + e^Y = e^{x+y}$, prove that $\frac{dy}{dx} + e^{Y-X}=0$
3. If x=a(cos t + log tan $\frac{t}{2}$) , y= a sin t, then evaluate $\frac{d2y}{dx2}$ at t= $\frac{\pi}{3}$.
4. If y=Pe^{ax} + Qe^{bx}, then show that d^2y/dx^2 –(a+b)$\frac{dy}{dx}$ = aby=0.
5. If x= cos*t*(3- 2 cos^{2}t) and y= sin*t* (3-2sin^{2}t), then find the value of $\frac{dy}{dx}$ at t=$\frac{\pi}{4}$.
6. If(tan^{-1}x)^y + y$^{\cot x}$=1, then find *dy/dx.*
7. If $x^m y^n= (x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.
8. If $y = \sin^{-1}\{x\sqrt{1-x}-\sqrt{1-x^2}\}$ and 0 < x < 1, then $\frac{dy}{dx}$,
9. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx}+e^{y-x}=0$.
10. If $x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a$ sint, then evaluate $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$.
11. Differentiate tan^{-1} $\left(\frac{x}{\sqrt{1-x^2}}\right)$ w.r.t sin^{-1} $\left(2x\sqrt{1-x^2}\right)$.
12. If y = Pe^{ax} + Qe^{bx}, then show that $\frac{d^2y}{dx^2}-(a+b)\frac{dy}{dx}+aby=0$.
13. If x = cost (3-2 cos^{2}t), then find the value of $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$.
14. If x a (cos t + t sin t) and y = a (sin t – t cos t), then find the value of $\frac{d^2y}{dx^2}$ at t $\frac{\pi}{4}$.
15. If y = tan^{-1} $\left(\frac{a}{x}\right)+\log\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx}=\frac{2a^2}{x^4-a^4}$.
16. If x = a sin 2 t (1-cos2t) and y = b cos2t (1-cost), then show that at $t=\frac{\pi}{4}, \left(\frac{dy}{dx}\right)=\frac{b}{a}$.
17. If (tan^{-1}x)^y + y$^{\cot x}$ = 1, then find dy/dx.

4 mark Questions

1. If f(x)= $\begin{cases} \frac{1-\cos 4x}{x2}, when\ x<0 \\ a, \quad when\ x=0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}}, when\ x>0 \end{cases}$

   And f is continuous at x=0, then find the value of a.
2. Find the value of k, so that the function f defined by

   f(x)=$\begin{cases} \frac{k\cos x,}{\pi-2x}\ if\ x\neq\pi 2 \\ 3,\ \ if\ x=\pi/2 \end{cases}$is continuous at x=$\frac{\pi}{2}$.
3. If the function f(x) given by

$$f(x)=\begin{cases}3ax+b, if\ x>1\\ 11, if\ x=1\\ 5ax-2b, if\ x<1\end{cases}$$

is continuous at x=1, then find the value of a and b.

4. Find the values of a and b such that the following function f(x) is a continuous function.

$$f(x)=\begin{cases}kx+1, if\ x\le \pi\\ \cos x, if\ x>\pi\end{cases}$$

is continuous at x=π

5. Discuss the continuity of the function f(X) at x=1/2, when f(x) is defined as follows:

$$f(x)=\begin{cases}\frac{1}{2}+x, 0\le x\le 1/2\\ 1, x=1/2\\ \frac{3}{2}+x, \frac{1}{2}<x\le 1\end{cases}$$

6. Find the values of a and b such that the function defined as follows is continuous.

$$f(x)=\begin{cases}x+2, x\le 2\\ ax+b, 2<x<5\\ 3x-2, x\ge 5\end{cases}$$

7. Shoe that $f(x)=\begin{cases}5x-4, when\ 0<x\le 1\\ 4x3-3x, when\ 1<x<2\end{cases}$

Is continuous at x=1.

8. If $(\tan^{-1}x)^y + y^{\cot x} = 1$, then find dy/dx.
9. If y $(\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.
10. If y = log $\left[x+\sqrt{x^2+a^2}\right]$, then show that $(a^2+b^2)\frac{d^2y}{dx^2}= x\frac{dy}{dx}=0$.
11. Differentiate the following with respect to x $\sin^{-1}\left[\frac{2^{x+1}.3^x}{1+(36)^x}\right]$.
12. if x sin (a+y) + sin a cos (a+y) =0, then prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$.
13. If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$ or $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}x^2}$.
14. If sin y = x sin (a+y), then prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$.
15. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ w.r.t.x.
16. If y = $(\tan^{-1}x)^2$, then show that $(x^2+1)^2\ (x^2+1)\frac{dy}{dx}=2$.
17. Find $\frac{dy}{dx}$, when $y=x^{\cot x}+\frac{2x^2-3}{x^2+x+2}$.
18. If x $\sqrt{1+y}+y\sqrt{1+x}=0, (x\neq y)$, then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.
19. If x = tan $\left(\frac{1}{a}\log y\right)$, then show that $(1+x^2)\frac{d^2y}{dx^2}+\frac{(2x-a)dy}{dx}=0$.
20. Prove that
21. $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]=\sqrt{a^2-x^2}$.
22. If log $\left(\sqrt{1+x^2}-x\right)=y\sqrt{1+x^2}$, then show that $(1+x^2)\frac{dy}{dx}$ + xy + 1 = 0.
23. If x = a (cosθ + θ sinθ) and y = a (sinθ - θ cosθ), then find $\frac{d^2y}{dx^2}$.

24. If $y = \cos^{-1}\left[\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right]$, then find $\frac{dy}{dx}$.

25. Show that the function defined as follows, is continuous at x = 1, x = 2 but not differentiable at x =2

26. $f(x) = \begin{cases} 3x-2, 0 < x \le 1 \\ 2x^2 - x, 1 < x \le 2 \\ 5x-4, x > 2 \end{cases}$

27. If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.

28. If y = 3 cos (log x) + 4 sin (log x), then show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

29. Differentiate the following function w.r.t.x. $x^{\sin x} + (\sin x)^{\cos x}$.

30. If $y = \cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, $0 < x < \frac{\pi}{2}$, then find $\frac{dy}{dx}$.

31. Differentiate the following function w.r.t.x.

32. $\tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$.

Chapter 6

Application of Derivatives

1 mark Questions

1. The total cost C(x) associated with provision of free mid-day meals to x students of a school in primary classes is given by
2. $C(x) + 0.005x^3 - 0.02x^2 + 30x + 50$
3. If the marginal cost is given by rate of change $\frac{dC}{dx}$ of total cost, then write the marginal cost of food for 300 students. What value is shown here?
4. Find the slope of tangent to the curve $y = 3x^2 - 6$ at the point on it whose x-coordinate is 2.
5. Find the slope of the tangent to the curve $y = 3x^2 - 4x$ at x=1.
6. For the curve $y = 3x^2 + 4x$, find the slope of tangent to the curve at point, where x-coordinate is -2.

4 marks Questions

1. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is
    i) Strictly increasing
    ii) Strictly decreasing
2. Using differentials, find the approximate value of $(3.986)^{3/2}$.
3. Find the approximate value of f(3.02), upto 2 places of decimal, where $f(x) = 3x^2 + 5x + 3$.
4. A ladder 5m long is leaning against wall. Bottom of ladder is pulled along the ground away from the wall at the rate of 2m/s. How fast is the height on the wall decreasing, when the foot of ladder is 4m away from the wall?
5. Find the intervals in which function given by $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is
    i) Increasing
    ii) Decreasing
6. If the radius of sphere is measured as 9 cm with an error of 0.03 cm, then find approximate error in calculating its surface area.
7. Find the intervals in which the function f(x) = sin x + cos x, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.
8. Find the intervals in which the function
    $F(x) = 2x^3 - 9x^2 + 12x + 15$ is
    i) Increasing
    ii) Decreasing
9. The length x of a rectangle is decreasing at the rate of 5cm/min and the width y s increasing at the rate of 4 cm/min. when x=8 cm and y= 6 cm, find the rate of change of
    i) Perimeter
    ii) Are of rectangle

10. If $f(X) = 3x^2 + 15x + 5$, then find the approximate value of f (3.02) using differentials.

11. Find the points on the curve $y = x^3-11x+5$ at which equation of tangent is $y= x-11$.

12. Find the equation of the tangent to curve $x=\sin 3t$, $y = \cos 2t$ at $t=\frac{\pi}{4}$.

13. Find the equation of tangent to curve $y= \frac{x-7}{x2-5x+6}$ at the point, where it cuts x-axis.

14. At which points will the tangent to the curve $y= 2x^3 - 15x^2 + 36x -21$ be parallel to X-axis? Also, find the equations of tangents to the curve.

15. Find the equation of tangent to the curve $y = \sqrt{3}x-2$, which is parallel to the line 4x- 2y +5 +0.

16. The sides of an equilateral triangle re increasing at the rate of 2cm/s. find the rate at which the area increases, when the side is 10cm.

17. The sum of the perimeters of a circle and square is k, where k is some constant. Prove that the sum of their areas is least, when the side of the square is double the radius of the circle.

18. Find the equation of tangent to the curve $4x^2 + 9y^2= 36$ at the point (3 cos ∅, 2 sin ∅).

6 marks Questions

1. Prove that the function f is defined by $f(x) = x^2 -x+ 1$ is neither increasing nor decreasing in (-1, 1). Hence, find the intervals in which f(x) is
2. Strictly increasing
3. Strictly decreasing
4. Find the intervals in which the function f given by f(X) = sin x – cos x, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.
5. Find the equations of the tangent to the curves $y= x^2 - 2x+ 7$ which is
6. Parallel to the line 2x-y+9 +0
7. Perpendicular to the line
    1. 5 y- 15 x + 13.
8. Find the equation of the normal at a point on the curves $x^2 + 4y$, which passes through the point (1,2). Also, find the equation of the corresponding tangent.
9. For the curve $y= 4x^3- 2x^5$, find all the points on the curve at which the tangent passes through the origin.
10. Find the equations of tangent and normal to the curve x=1- cos Θ, y = Θ- sin Θ at $Θ=\frac{\pi}{4}$.
11. If the lengths of three sides of a trapezium other than the base are equal to 10cm, then find the area of trapezium, when it is maximum.
12. Find the point p on the curve $y^2+ 4ax$, which is nearest to the point (11 a, 0).
13. Of all the closed right circular cylindrical cans of volume 128 π cm^3, find the dimension of the can which has minimum surface area.

14. Prove that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.
15. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.
16. An open box with a square base is to be made out of a given quantity of cardboard of area C^2 sq. units. Show that the maximum volume of box is $\frac{C3}{6\sqrt{3}}$ cu units.
17. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.
18. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m. Then find the dimensions of the rectangle that will produce the largest area of the window.
19. Show that the semi vertical angle of a right circular cone of maximum volume and given slant height $\tan^{-1}\sqrt{2}$.
20. Find the point on the curve $y^2 + 2x$ which is at a minimum distance from the point (1, 4).
21. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.
22. A tank with rectangular base and rectangular sides, open at the top is to be constructed, so that its depth is 2m and volume is $8m^3$. If building of tank cost Rs 70 per sq. m for the base and Rs 45 per sq m for sides. What is the cost of least expensive tank?
23. Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height 1/3 h.
24. Show that the right circular cylinder, open at the top and of given surface area and maximum volume is such that its height is equal to the radius of the base.

Chapter 7

Integrals

1 mark Questions

1. Write the anti-derivative of $(3\sqrt{x}+\frac{1}{\sqrt{x}})$..
2. Evaluate $\int (1-x)\sqrt{x}dx$.
3. Given, $\int e^x(\tan x+1) \sec x dx= e^x f(x)+c$. Write f(x) satisfying above.
4. Evaluate $\int \sec^2(7-4x)dx$
5. Evaluate $\int \sin^3 x dx$.
6. Evaluate $\int \frac{dx}{\sin^2 x\cos^2 x}$.
7. Write the anti-derivative of $\left(3\sqrt{x}+\frac{1}{\sqrt{x}}\right)$.
8. Evaluate $\int(1-x)\sqrt{x}dx$.
9. Given, $\int e^2 (\tan x + 1) \sec x dx = e^2 f(x) + C$. Write f(x) satisfying above.
10. Write the value of $\int \frac{x+\cos 6x}{3x^2+\sin 6x}dx$.
11. Write the value of $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$.
12. Evaluate $\int \frac{(\log x)^2}{x}dx$.
13. Evaluate $\int \frac{2 \cos x}{3\sin^2 x}dx$.
14. Evaluate $\int \sin^2 (7-4x)$ dx.
15. Evaluate $\int \frac{\log(\sin x)}{\tan x}dx$.
16. Evaluate $\int \frac{\sec^2 x}{3+\tan^2 x}dx$.
17. Evaluate $\int \frac{x^2}{3+\tan^2 x}dx$.
18. Evaluate $\int \sin^2 x dx$.
19. $\int_2^3 \frac{1}{x}dx$
20. $\int_0^1(3x^2+2x+k)dx$, then find the value of k.
21. $\int_0^{\pi/4} sin2xdx$
22. $\int_{-\pi/4}^{\pi/4}(\text{Sin}^3x\ dx)$

4 mark Questions

1. Evaluate $\int(x-3)\sqrt{x^2+3x-18}dx$.
2. Evaluate $\int \frac{x+2}{\sqrt{x^2+5x+6}}$ dx.
3. Find $\int \frac{5x-2}{1+2x+3x^2}dx$.

4. Evaluate $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$.

5. Evaluate $\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$.

6. Evaluate $\int \frac{x^2}{(x^2+4)(x^2+9)}dx$.

7. Evaluate $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$.

8. Evaluate $\int \frac{dx}{x(x^5+3)}$.

9. Evaluate $\int \sin x.\sin^2 x.\sin 3x dx$.

10. Evaluate $\int \frac{x^2}{(x\sin x+\cos x)^2}dx$.

11. Evaluate $\int \frac{x^2+1}{x^4+1}dx$.

12. Evaluate $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$.

13. Evaluate $\int \left[\log(\log x)+\frac{1}{\log^2}\right]dx$.

14. Evaluate $\int \frac{x+2}{\sqrt{(x-2)(c-3)}}dx$.

15. Evaluate $\int \frac{(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$.

16. Evaluate $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

17. Evaluate $\int x\sin^{-1}x dx$.

18. Evaluate $\int x.\log|(x+1)|dx$.

19. Evaluate $\int_0^{\pi/4} \log(1+tanx)\,dx$

20. Evaluate, $\int_0^4 (|x| + |x-2| + |x-3|)dx$

21. Evaluate $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}$ dx

22. Evaluate $\int_0^{\pi/2} \log\sin x\, dx$

23. Evaluate, $\int_0^1 \log(\frac{1}{x-1})$

24. Evaluate, $\int_{-1}^2 |x^3-x|dx$

6 mark Questions

1. Evaluate $\int(\sqrt{\cot x}+\sqrt{\tan x})dx$.

2. Evaluate $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx, x\in[0,1]$.

3. Find $\int \frac{\sqrt{x^2+1}\,(\log(x^2+1)-2\log x)}{x^4}dx$.

4. Evaluate $\int \frac{x^2+1)}{(x-1)^2(x+3)}dx$.

5. Evaluate $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx$.

6. Evaluate $\int_1^3 (2x^2+5x)dx$ as a limit of a sum.

7. Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x.cosec\, x}$ dx

8. Evaluate, $\int_0^1 \cot^{-1}(1-x+x^2)\,dx$

9. Evaluate $\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

10. Evaluate $\int_0^2 (x^2-x)\,dx$ as a limit of sum.

Chapter 8

Application of Integrals

6 marks Questions

1. Using integration. Find the area of the region bounded by the curves.
2. $y = |x-1| + 1$, $x= -3$, $x=3$ and $y =0$.
3. Using integration, find the area of the region bounded by the triangle whose vertices are (-1,2), (1,5) and (3,4).
4. Using method of integration, find the area of the region bounded by lines $2x+y=4$, $3x-2y=6$ and $x-3y+5 +0$.
5. Using integration, find the area bounded by the curve $x^2= 4y$ and the line $x= 4y -2$.
6. Using integration, find the area of the region enclosed between the two circles
7. $X^2 + y^2= 4$ and $(x-2)^2 + y^2= 4$
8. Find the area of the region $\{(x,y):y^2\leq 4x,\ 4x^2+ 4y^2\leq 9\}$, using method of integration.
9. Find the area of the region bounded by the parabola $y=x^2$ and $y=|x|$.
10. Find the area of the region given by $\{(x,y): x^2\leq y\leq |x|\}$.
11. Using integration, find the area of the triangular region whose sides have the equations $y=2x+1$, $y=3x+1$ and $x=4$.
12. Find the area of the region
13. $\{(x,y): (x^2+y^2) \leq 1\leq x+y\}$
14. Using integration, find the area of ΔABC, the coordinates of whose vertices are A(2,5), B(4,7) AND C(6,2).
15. Find the area of circle $4x^2+ 4y^2 =9$ which is interior to the parabola $x^2=4y$.
16. Using integration, find the area of the region bounded by the triangle whose vertices are (1,3), (2,5) and (3,4).
17. Find the area of the region bounded by the region enclosed by the curves
18. $(x-6)^2 + y^2 =36$ and $x^2 + y^2=36$.
19. Find the area of the region bounded by the parabolas $y^y= 4ax$ and $x^2= 4ay$.
20. Find the area of the region enclosed between two circles $x^2 + y^2 =9$ and $(x-3)^2 + y^2=9$.

Chapter 9

Differential equations

1 mark Questions

1. Write the order and degree of $\frac{dy}{dx}$+ cos y=0
2. Write the degree of the differential equation

   $x^3\left(\frac{d2y}{dx2}\right)^2 + x\left(\frac{dy}{dx}\right)^{4=0}$
3. Write the order and degree of $\frac{dy}{dx} + tan\left(\frac{dy}{dx}\right)$=0
4. Write the degree of differential equation

   $\left(\frac{dy}{dx}\right)^4 + 3d^2y/dx^2=0$
5. Write the differential equation representing the family of curves y=mx, where m is an arbitrary constant.

4 marks Questions

1. Find the differential equations of family of circle touching Y-axis at the origin.
2. Find the differential equation f family of circles touching X-axis at the origin.
3. Form the differential equation representing family of ellipses having foci X-axis and centre at the origin.
4. Find the particular solution of the differential equation x$\frac{dy}{dx}$ – y cosec$\left(\frac{y}{x}\right)$=0 or $\frac{dy}{dx}$ -$\frac{y}{x}$+ cosec $\left(\frac{y}{x}\right)$=0, given that y=0, when x=1.
5. Solve the differential equations
6. xlog x$\frac{dy}{dx}$ +y + $\frac{2}{x}$ log x.
7. Find the particular solution f the differential equation
8. $\frac{dy}{dx} = \frac{x(2logx+1)}{\sin y+y\cos y}$, given by y=$\frac{\pi}{2}$, when x=1.
9. Find the particular solution of the differential equation $x(1+y^2)dx- y(1+x^2)dy=0$, given that y=1, when x=0
10. Find the particular solution of the differential equation log$\left(\frac{dy}{dx}\right)$= 3x+ 4y equation, given that y=0, when x=0
11. Find the differential equation of an ellipse with major and minor axes 2a and 2b respectively.
12. Form the differential equation representing the family of curves$(y– b)^2= 4(x– a)$.
13. If y(x) is a solution of differential equation $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx}$ = -cos x and y(0)=1, then find the value of y$\left(\frac{\pi}{2}\right)$.
14. Solve, $\frac{dy}{dx}$+ y sec x= tan x
15. Find the particular solution of the differential equation

16. $(1+ e^{2x})dy + (1+y^2)e^x dx=0$, given that y=1, when x=0.
17. Solve the following differential equation
18. $(y+3x^2)\frac{dy}{dx} = x$
19. Solve the following differential equation
20. $(1+y^2)(1+\log x)\, dx + x dy=0$
21. Show that the following differential equation is homogenous and then solve it
22. $y\, dx + x \log\left(\frac{y}{x}\right) dy- 2x\, dy=0$
23. Solve the following differential equation
24. $\cos^2 x\frac{dy}{dx} + y= \tan x$
25. Solve, $(x \log x)\frac{dy}{dx} +y= 2\log x$
26. Solve, $\frac{dy}{dx} +y= \cos x -\sin x$
27. Solve, $\frac{dy}{dx}= \frac{x(2y-x)}{x(2y+x)}$, if y=1, when x=1.

6 marks Questions

1. Show that the differential equation $2ye^{x/y} dx + (y-2xe^{x/y})dy=0$ is homogenous. Find the particular solution of this differential equation, given that x=0, when y=1.
2. Find the particular solution of the differential equation $(\tan^{-1} y-x)dy= (1+y^2)dx$, given that x=0, when y=0.
3. Show that the differential equation

   $[x \sin^2(\frac{y}{x}) -y]dx+ xdy=0$ is homogenous. Find the particular solution of this differential equation, given that $y=\frac{\pi}{4}$, when x=1.
4. Solve, $(x\, dy- y\, dx)\, y \sin\left(\frac{y}{x}\right)= (y\, dx + x\, dy)\; x \cos\left(\frac{y}{x}\right)$
5. Solve, $\frac{dy}{dx} + y \cot x= 2x + x^2 \cot x$, given that y(o)=0

Chapter 10

Vector Algebra

1 mark Questions

1. Find a vector $\underset{a}{\rightarrow}$ of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with X-axix, $\frac{\pi}{4}$ with Y-axis and an acute angle θ with Z-axis.
2. Write the value of the following:
$\overrightarrow{PQ}$, where $\vec{P}$ and $\vec{Q}$ are the points (1, 3, 0) and (4, 5, 6) respectively.
3. If $\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\hat{\imath} + k$ are two equal vectors, then write the value of x + y + z.
4. L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$, respectively. Write divides the line segment LM in the ration 2 : 1 externally.
5. Find the sum of the vectors $\vec{a}$ = $\hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = -2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = \hat{\imath} - 6\hat{\jmath} - 7\hat{k}$.
6. Find the scalar components of $\overrightarrow{AB}$ with initial point A (2, 1) and terminal point B (-5, 7).
7. Write a unit vector in the direction of vector $\vec{a}$=$2\hat{\imath} + \hat{\jmath} + 2\hat{k}$.
8. Find a vector in the direction of $\vec{a}$=$2\hat{\imath} - \hat{\jmath} + 2\hat{k}$, which has magnitude 6 units.
9. Find a position vector of mid-point of the line segment AB, where A is point (3, 4, -2) and B is point (1, 2, 4).
10. Write a vector of magnitude 9 units in the direction of vector - $2\hat{\imath} + \hat{\jmath} + 2\hat{k}$.
11. What is the cosine of angle which the vector $\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k}$ makes with Y-axis?
12. Find a unit vector in the direction of vector $\vec{a}$=$2\hat{\imath} - \hat{\jmath} + 2\hat{k}$.
13. Find the magnitude of the vector $\vec{a}$=$2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$.
14. If $\vec{a}$=$\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b}$=$3\hat{\imath} + \hat{\jmath} - 2\hat{k}$, then find a unit vector in the direction$\vec{a} - \vec{b}$.
15. Find the projection of vector $\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$ on the vector $2\hat{\jmath} + 6\hat{k}$.
16. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, find the angle between $\vec{b}$ and $\vec{b}$.
17. Write the value of λ, so that the vectors $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ are perpendicular to each other.
18. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.
19. Find λ, when projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.
20. If $\vec{a}.\vec{a} = 0$ and $\vec{a}.\vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?
21. If $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60^0, then find $\vec{a}.\vec{b}$.
22. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 3$, then find the projection of $\vec{b}$ on $\vec{a}$.
23. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$, then find the angle between $\vec{a} \times\times \vec{b}$.
24. Find $\vec{a}.\vec{b}$, if $\vec{a} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$.
25. If $\hat{p}$ is a unit vector and $(\vec{x} - \hat{p}).(\vec{x} + \hat{p})$ = 80, then find $|\vec{x}|$.
26. Write the value of P, for which $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + P\hat{\jmath} + 3\hat{k}$ are parallel vectors.

27. Find value of the following: $\hat{\imath}.(\hat{\jmath}\times\hat{k})+\hat{\jmath}(\hat{\imath}\times\hat{k})+\hat{k}.(\hat{\imath}\times\hat{\jmath})$.
28. If $|\vec{a}|=\sqrt{3},|\vec{b}|=2$ and $\vec{a}.\vec{b}=3$, then find the angle between $\vec{a}$ and $\vec{b}$.
29. Find angle between vectors $\vec{a}=\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}+\hat{\jmath}-\hat{k}$.

4 mark Questions

1. Let $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k},\vec{b}=4\hat{\imath}-2\hat{\jmath}+3\hat{k}$ and $\vec{c}=\hat{\imath}-2\hat{\jmath}+\hat{k}$. Find a vector of magnitude 6 units, which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.
2. Find the position vector of a point R, which divides the line joining two points P and Q whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively, externally in the ration 1 : 2. Also, show that P is the mid-point of line segment RQ.
3. Vectors $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3,|\vec{b}|$=5 and $|\vec{c}|=7$. Find the angle between $\vec{a}$ and $\vec{b}$.
4. The scalar product of the vector $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{c}$=$\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is equal to one. Find the value of λ and hence, find the unit vector along $\vec{b}+\vec{c}$.
5. Find the unit vector perpendicular to the plane ABC where the position vectors of A, B and C are 2$\hat{\imath}$ -$\hat{\jmath}$+$\hat{k}$, $\hat{\imath}+\hat{\jmath}$+2$\hat{k}$ and 2$\hat{\imath}$+3$\hat{k}$ respectively.
6. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of the same magnitude then prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with the vectors $\vec{a},\vec{b}$ and $\vec{c}$.
7. If $\vec{a}=\hat{\imath}+\hat{k}$ and $\vec{b}=\hat{\jmath}-\hat{k}$, then find a vector $\vec{c}$, such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}$ =3.
8. If $\vec{p}=5\hat{\imath}+\lambda\hat{\jmath}-3\hat{k}$ and $\vec{q}=\hat{\imath}+3\hat{\jmath}-5\hat{k}$, then find the value of , so that $\vec{p}+\vec{q}$ and $\vec{p}-\vec{q}$ are perpendicular vectors.
9. Let $\vec{a}=\hat{\imath}+4\hat{\jmath}+2\hat{k},\vec{b}=3\hat{\imath}-2\hat{\jmath}+7\hat{k}$ and $\vec{c}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c}=18$.
10. If vectors $\vec{a}=2\hat{\imath}+2\hat{\jmath}+3\hat{k},\vec{b}=-\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\vec{c}=3\hat{\imath}+\hat{\jmath}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.
11. If $\vec{a},\vec{b}$ and $\vec{c}$ are three vectors, such that $|\vec{a}|=3,|\vec{b}|=4$ and $|\vec{c}|=5$ and each one of these is perpendicular to the sum of other two, then find $|\vec{a}+\vec{b}+\vec{c}|$.
12. The scalar product of vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\lambda\hat{\imath}$+2j $+3\hat{k}$ is equal to one. Find the value of λ.
13. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, the show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.
14. Find a vector of magnitude 5 units, perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$.

Chapter 11

Three Dimensional Geometry

1mark Questions

1. What is the distance of point (a, b, c) from x-axis?
2. What is the angle between the lines 2x= 3y= – zand 6x = – y = – 4z.
3. Find the direction cosines of the lines

   $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$
4. What are the direction cosines of a line whch makes equal angles with the coordinate axes?

   Find the Cartesian equation of the line which passes through the point (-2, 4,-5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$
5. Find the distance of the point (2,3,4) from X-axis?
6. Find the angle between the planes 2x– 3y+ 6z= 9 and xy– plane
7. Write the direction cosines of the line parallel to the line

   $\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$
8. If P=(1,5,4) and Q=(4,1,-2), then find the direction ratios of PQ.
9. If the direction ratios of a line are (1, –2, 2) then what are the direction cosines of the line.
10. What is the equation of the plane through the point (1, 4, – 2) and parallel to the plane – 2x+ y– 3z= 7
11. Find the length of the perpendicular drawn from the origin to the plane 2x-3y+6x+21=0
12. What is the perpendicular distance of plane 2x– y+ 3z= 10 from origin.
13. What is the y-intercept of the plane x– 5y+ 7z= 10
14. Write the distance of following plane from the origin, 3x-y+2z+1=1.

    Find the value of λ, such that the line $\frac{x-2}{\lambda} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$, is perpendicular to the plane 3x-y-2z=7
15. Are the planes x+ y– 2z+ 4 = 0 and 3x+ 3y– 6z+ 5 = 0 intersect

4 marks Questions

1. Find the direction cosines of the line

   $\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{-6}$. Also, find the vector equation of the line through the point A(-1,2,3) and parallel to the given line.
2. Find the value of p, so that the lines

   $l_1 = \frac{1-X}{3} = \frac{7Y-14}{P} = \frac{Z-3}{2}$

   And $l_2 = \frac{7-7X}{3P} = \frac{Y-5}{1} = \frac{6-z}{5}$

   are perpendicular to each other, Also, find the equation of a line passing through a point (3,-2,4) and parallel to line l_2
3. Find the shortest distance between the following lines

$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$, $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

4. The Cartesian equation of a line 6x-2=3y+1= 2z-2. Find the direction cosines of the line. Write down the Cartesian and vector equations of a line passing through (2,-1,-1) which are parallel to the given line.
5. Find the angle between following pair of lines

   $\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3}$ and

   $\frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$

   and check whether the lines are parallel or perpendicular.
6. Find the length and foot of perpendicular drawn from the point (2,-1,5) to line

   $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$
7. Find the value of λ, so that the lines

   $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ and $\frac{7-7x}{\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$
8. Find the equation of the perpendicular distance of point (1,0,0) from the lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$. Also, find the coordinates of foot of perpendicular and equation of perpendicular.
9. Find the image of the point having position vector $\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ in the plane $\vec{r}(2\hat{\imath} + \hat{\jmath} + \hat{k}) + 3 = 0$.
10. Find the coordinates of the point, where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersects the plane x-y+z-5=0. Also, find the angle between the line and the plane.
11. Find the equation of plane(s) passing through the intersection of planes x+3y+6=0 and 3x-y-4z=0 and whose perpendicular distance from the origin is unity.
12. Find the Cartesian equation of the plane passing through points A(0,0,0) and B(3,-1,2) and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$
13. Find the equation of the plane that contains the point (1,-1,2) and is perpendicular to each of planes 2x+3y-2z=5 and x+2y-3z=8.

6 marks Questions

1. Find the distance of the point P(-1,-5,-10) from the point of intersection of the line joining the points A(2, -1,2) and B(5,3,4) with the planes x-y+z=5.
2. Find the equation of line passing through points A (0,6,-9) and B (-3,-6,3). If D is the foot of perpendicular drawn from the point C (7,4,-1) on the line AB, then find the coordinates of point D and equation of line CD.
3. The points a(4,5,10), B (2,3,4) and C(1,2,-1) are three vertices of parallelogram ABCD. Find the vector equations of sides AB and BC and also find coordinates of point D.
4. Find the perpendicular distance of point (2,3,4) from the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, find coordinates of foot of perpendicular.

5. Find the equation of the plane through the line of intersection of the planes x+y+z=1 and 2x+3y+4z=5, which is perpendicular to the plane x-y+x=0. Also, find the distance of the plane obtained above, form the origin.

6. Find the equation of plane determined by points A(3,-1,20, B(5,2,4) and C(-1,1,6) and hence find the distance between planes and point (6,5,9).

7. Find the length of the foot of perpendicular from the point P(7,14,5) to plane 2x+4y-z=2. Also, find the image of point P in the plane.

8. Find the equation of plane passing through the point (-1,3,2) and perpendicular to each of the planes x+2y+3z=5 and 3x+3y+z=5.

9. Find the vector equation of plane passing through the points A (2,2,-1), B(3,4,2) and C(7,0,6). Also, find the Cartesian equation of the plane.

10. Find the coordinates of the foot of perpendicular and the perpendicular distance of point P (3,2,1) from the plane 2x-y+z+1=0. Also, find image of the point in the plane.

11. Find the distance of point (-2,3,-4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane 4x+12y-3z+1=0.

12. Find the equation of plane passing through the point (-1,1,2) and perpendicular to each plane 2x+3y-3z=2 and 5x-4y+z=6.

Chapter 12

Linear Programming

6 marks Questions

1. I f a young man rides his motor-cycle at 25km per hour; he had to spend of 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to Rs. 5 per km and rate of pollution also increases. He has Rs.100 to spend on petrol and wishes to find the maximum distance he can travel in one hour. Express this problem as an LPP. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?
2. One kind of cake requires 200 g of flour and 25g of fat, another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an LPP and solve it graphically.
3. A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760 to invest and has space for almost 20 items for storage. An electronic sewing machine cost him Rs. 360 and a manually operated sewing machine Rs. 240. He can sell an electronic sewing machine at a profit of rs.22 and a manually operated sewing machine at a profit of rs.18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as an LPP and solve it graphically.
4. A manufacturer produces nuts and bolts. It takes 1hr of work on machine A and 3hrs on machine B to produce package of nuts. It takes 3hr of work on machine A and 1hr on machine B to produce package of bolts. He earns a profit of Rs.17.50 per package on nuts and rs7 per package on bolts. How many packages of each should be produced each day so as to maximize his profits, if he operates his machines for almost 12hr a day? Formulate above as a LPP and solve it graphically.
5. A dietician wishes to mix two types of foods in such a way that the vitamin contents of mixture contains at least 8 units of vitamin A and 10 units of Vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C, while food II contains 1 unit per kg of vitamin A and 2units per kg of Vitamin C. it costs rs.5 per kg to purchase food II. Find the minimum cost of such a mixture. Formulate above as an LPP and solve it graphically.
6. A library has to accommodate two different types of books on a shelf. The books are 6cm and 4cm thick and weight 1kg and $1\frac{1}{2}$ kg each, respectively. The shelf is 96cm long and at most can support a weight of 21kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an LPP and solve it graphically.
7. Two tailors A and B earn rs.150 and rs.200 per day, respectively. A can stitch 6 shirts and 4 pants per day, while B can stich 10shirts and 4 pants per day. How many days shall each work, if it is desired to produce at least 60 shirts and 32 pants at a minimum labor cost? Make it as an LPP and solve the problem graphically.

8. A factory owner purchases two types of machine A and B for his factory. The requirements and the limitations for the machines are as follows:

9. Machines	10. Area occupied	11. Labor force	12. Daily output (in units)
13. A	14. 1000 m^2	15. 12 men	16. 60
17. B	18. 1200 m^2	19. 8 men	20. 40

Chapter 13

Probability

4 marks Questions

1. A couple has 2 children. Find the probability that both are boys, if it is known that
   a. One of them is a boy
   b. The older child is a boy
2. A speaks truth in 75% of the cases, while B in 90% of the cases. In what percent of cases, are they likely to contradict each other in stating the same fact? Do you think that statement of B is true?
3. In a hockey match, both teams A and B scored same number of goals upto the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, then find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.
4. 12 cards numbered 1 to 12 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, then find the probability that it is an even number.
5. Probabilities of solving a specific problem independently by A and B are ½ and 1/3, respectively. If both try to solve problem independently, find the probability that
   a. Problem is solved
   b. Exactly one of them solves the problem
6. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least three successes.
7. Out of a group of 30 honest people, 20 always speak the truth. Two persons are selected at random from the group. Find the probability distribution of the number of selected persons who speak the truth. Also, find the mean of the distribution. What values are described in this question
8. How many times must a man toss a fair coin, so that the probability of having atleast one head is more than 80%?
9. Find the probability distribution of number of doublets in three tosses of a pair of dice?
10. Two cards are drawn at successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of number of aces.
11. In a MCQ exam with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?
12. A pair of dice is thrown 4 times. If getting a doublet is a success, then find the probability distribution of number of successes.

13. A card from a pack of 52 cards is lost. From the remaining cards of the pack, three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the four lost cards being a spade.

14. A random variable X has following probability distributions:

15. X	16. 0	17. 1	18. 2	19. 3	20. 4	21. 5	22. 6	23. 7
24. P(x)	25. 0	26. k	27. 2k	28. 2k	29. 3k	30. K^2	31. $2K^2$	32. $7k^2 +k$

33. Find (i) k (ii) P(X<3) (iii) P(X>6) (iv) P(0<X<3)

6 marks Questions

1. In a game, a man wins rupees five for a six and loses rupee one for any other number. When a fair dice is thrown, he decides to throw a dice first but to quit as when he gets a six. Find the expected value of amount he wins/loses.
2. An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the balls in the urn are white?
3. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident for them are 0.01, 0.03, and 0.15 respectively.
4. One of the insured persons meets with an accident. What is the probability that he is a scooter driver or car driver?
5. Among the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostels). Previous year's results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual exams. At the end of year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostelier?
6. Suppose 5% of mean and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.
7. Bag 1 contains 3 red and 4 black balls and bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and is found to be ed. Find the probability that it was drawn from the bag II.
8. A man is known to speak truth 3 out of 4 times. He throws a dice and reports that it is a six. Find the probability that it is actually a six.
9. There are three coins. One is two tailed coin another is a biased coin that comes up heads 60% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed and it shows tail. What is the probability that it is a two tailed con?

10. A bag contains 4 red and 4 black balls. Other bag contains 2 red and 6 black balls. One of the bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that ball drawn is from the first bag.

11. Three bags contain balls as shown in the table:

12. Bag	13. White Balls	14. Black Balls	15. Red Balls
16. I	17. 1	18. 2	19. 3
20. II	21. 2	22. 1	23. 1
24. III	25. 4	26. 3	27. 2

28. A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from bag III?

29. In a bulb factory, machines A, B, and C manufacture 60%, 30%, and 10% bulbs, respectively. 1%, 2% and 3% bulbs produced respectively are found to be defective. A bulb is picked at random from the total production and found to be defective. Find the probability that this bulb was produced by machine A.

30. Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head. What is the probability that she throws 1, 2, 3 or 4 with the dice?

31. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

32. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained?

33. If a fair coin is tossed 10 times, find the probability of getting.
    a. exactly six heads,
    b. at least six heads,
    c. at most six heads